

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\begin{array}{ll} \mathcal{L}[t^n] = \frac{n!}{s^{n+1}} & \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}t^{n-1} \\ \mathcal{L}[e^{at}] = \frac{1}{s-a} & \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at} \\ \mathcal{L}[\sin at] = \frac{a}{s^2+a^2} & \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at \\ \mathcal{L}[\cos at] = \frac{s}{s^2+a^2} & \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at \end{array}$$

First Differentiation Formula

$$\begin{aligned} \mathcal{L}[D^n x] &= s^n \mathcal{L}[x] - s^{n-1}x(0) - s^{n-2}x'(0) - \dots - x^{(n-1)}(0) \\ \mathcal{L}\left[\int_0^t f(u) du\right] &= \frac{1}{s}\mathcal{L}[f(t)] \quad \mathcal{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)] du \end{aligned}$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$, so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

$$\mathcal{L}[e^{at}f(t)] = F(s-a) \quad \mathcal{L}^{-1}[F(s)] = e^{at}\mathcal{L}^{-1}[F(s+a)]$$

Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} \mathcal{L}[f(t)] \quad \mathcal{L}^{-1}\left[\frac{d^n F(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\mathcal{L}[g(t+a)] \quad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$

Convolution

$$\mathcal{L}^{-1}[F(s)G(s)] = \mathcal{L}^{-1}[F(s)] * \mathcal{L}^{-1}[G(s)]$$

where

$$(f * g)(t) = \int_0^t f(t-u)g(u) du.$$

Convolutions of Trigonometric Functions

$$\begin{aligned} (\sin at) * (\cos at) &= \frac{t}{2} \sin at \\ (\sin at) * (\sin at) &= \frac{1}{2\alpha} \sin at - \frac{t}{2} \cos at \\ (\cos at) * (\cos at) &= \frac{1}{2\alpha} \sin at + \frac{t}{2} \cos at \end{aligned}$$