

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\begin{aligned} \mathcal{L}[t^n e^{\lambda t}] &= \frac{n!}{(s - \lambda)^{n+1}} & \mathcal{L}^{-1} \left[\frac{1}{(s - \lambda)^n} \right] &= \frac{1}{(n - 1)!} t^{n-1} e^{\lambda t} \\ \mathcal{L}[\sin at] &= \frac{a}{s^2 + a^2} & \mathcal{L}^{-1} \left[\frac{1}{s^2 + a^2} \right] &= \frac{1}{a} \sin at \\ \mathcal{L}[\cos at] &= \frac{s}{s^2 + a^2} & \mathcal{L}^{-1} \left[\frac{s}{s^2 + a^2} \right] &= \cos at \end{aligned}$$

First Differentiation Formula

$$\mathcal{L}[D^n x] = s^n \mathcal{L}[x] - s^{n-1} x(0) - s^{n-2} x'(0) - \dots - x^{(n-1)}(0)$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$, so $f(t) = \mathcal{L}^{-1}[F(s)]$

First Shift Formula

$$\mathcal{L}[e^{at} f(t)] = F(s - a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at} \mathcal{L}^{-1}[F(s + a)]$$

Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} \mathcal{L}[f(t)] \qquad \mathcal{L}^{-1} \left[\frac{d^n F(s)}{ds^n} \right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as} \mathcal{L}[g(t + a)] \qquad \mathcal{L}^{-1}[e^{-as} F(s)] = u_a(t)f(t - a)$$

Convolution

$$\mathcal{L}^{-1}[F(s)G(s)] = \mathcal{L}^{-1}[F(s)] * \mathcal{L}^{-1}[G(s)]$$

where
$$(f * g)(t) = \int_0^t f(t - u)g(u) du.$$

Convolutions of Trigonometric Functions

$$\begin{aligned} (\sin at) * (\cos at) &= \frac{t}{2} \sin at \\ (\sin at) * (\sin at) &= \frac{1}{2\alpha} \sin at - \frac{t}{2} \cos at \end{aligned}$$