

BLOCK: F+TF, Tue Fri 12:00-1:15 PM

INSTRUCTOR: Scott MacLachlan

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OFFICE: Bromfield-Pearson 212

OFFICE HOURS: (Fall 2008) TTh 9:30-11:15

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PREREQUISITES: Math 135 or consent.

TEXT: *Elementary Classical Analysis, Second Edition* by Jerrold E. Marsden and Michael J. Hoffman, W. H. Freeman and Company, 1993, New York.

#### COURSE DESCRIPTION:

In Math 136, you'll learn fundamental modern mathematics that is, by itself, beautiful, but is also very useful in fields as diverse as physics and economics. We will apply the tools we learned in Math 135 to develop the rigorous theory of differentiation and integration of functions on  $\mathbb{R}^n$ , and then we'll study Fourier series and Hilbert spaces.

Much of real analysis was developed to make the ideas of calculus rigorous and general. First, we will learn about the derivative of functions  $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ . This derivative is defined as a linear transformation (or a matrix); in Math 136, we'll explore what the entries should be in the derivative matrix. We will learn about the inverse and implicit function theorems, which are important in differential geometry and dynamical systems. The inverse function theorem gives conditions under which one can locally invert a function  $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$  (*answer: (essentially) when the derivative matrix is invertible*).

We will define the Riemann integral for functions  $f : A \rightarrow \mathbb{R}$  where  $A$  is a subset of  $\mathbb{R}^n$ . This will allow us to define the volume of many sets in  $\mathbb{R}^n$ . For example, the volume of the unit interval  $[0, 1] \subset \mathbb{R}$  is one, as you might guess. However, the set of rational numbers in  $[0, 1]$  has volume zero. We define a beautiful generalization, sets of measure zero (such as  $\mathbb{Q} \cap [0, 1]$ ), and use this concept to characterize the functions that have a Riemann integral. Then we will learn convergence theorems for integrals, such as Fubini's theorem which tells us why we can change the order of integration in a multiple integral.

Finally, we will learn some important tools of modern mathematics, Fourier Series and Hilbert Spaces. This will involve exciting general concepts, such as infinite dimensional vector spaces and infinite bases. We use these ideas to solve the partial differential equations describing light and heat and prove properties of the solutions.

Our goals are to lay the theoretical foundation of calculus and modern analysis and to hone your ability to formulate precisely mathematical ideas and to read and write proofs.