Real Analysis II

Math 136 | Course Information and Syllabus | Spring 2005

Block D+: Tuesday, Thursday 10:30-11:45

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Prerequisite: You need to have taken Math 135 but not necessarily in Fall 2004.

Text: *Elementary Classical Analysis,* **Second Edition** by Jerrold Marsden and Michael Hoffman.

Course Description: In Math 136, you will learn the mathematical theory of differentiation and integration of functions on $\mathbb{R}^n$, and you will refine your ability to do mathematical proofs. This mathematics is beautiful in its own right. It is also important in theoretical physics and advanced economics, and integration theory is a basis of probability. This course is one of the most exciting and most fundamental courses in pure mathematics.

Much of real analysis was developed to make the ideas of calculus rigorous and more general. First, we will learn about the derivative of functions $f : \mathbb{R}^n \to \mathbb{R}^m$. We will define the derivative as a linear operator (or matrix) and then prove theorems such as the Mean Value Theorem.

We will learn about the inverse and implicit function theorems. These theorems are fundamental to differential geometry. The inverse function theorem gives conditions when one can locally invert a function $f : \mathbb{R}^n \to \mathbb{R}^n$ (*answer: (essentially) when the derivative matrix is invertible*), and the implicit function tells when functions defined implicitly exist (*e.g.,* for which (x, y) does the equation $x^2+2xy+y^2 = 4$ define $y = y(x)$ as a differentiable function of x??).

We will define the Riemann integral for functions $f : A \to \mathbb{R}$ where A is a subset of $\mathbb{R}^n$. This will allow us to define the Riemann measure of many sets in $\mathbb{R}^n$. For example, the measure of the unit interval $[0, 1] \subset \mathbb{R}$ is one, as you might guess. However, the set of rational numbers is not Riemann measurable even though it is only countable. So, we introduce a new concept, (Lebesgue) measure zero, and $\mathbb{Q}$ has measure zero. Sets of measure zero are intriguing and they help characterize the functions that are Riemann integrable.

Then we will learn about convergence theorems for integrals. We'll learn why one can change the order of integration in a multiple integral (Fubini's Theorem). Finally, we will learn about L^2, orthogonal sets of functions, and the precise theory of Fourier Series. This will involve deep ideas from modern analysis, including Hilbert Spaces, complete inner product spaces. The most interesting will be infinite dimensional....with infinite bases. We plan to apply these ideas to solve partial differential equations.

You will gain a rigorous understanding of derivatives and integrals, and the basic ideas of multidimensional calculus will become mathematically clear and precise. I hope you will have a good time, and I know you will learn excellent mathematics.

Homework: Homework will be handed out about once a week (often on Thursdays) and graded. Homework is invaluable for your mastery of the course, and in order to do well, you must do homework. Homework (and class participation) will count for a substantial part of your grade. I plan to have help sessions on Wednesdays during the open block.

Exams: There will be two tests and a final in the course. *The second test will include an in-class part and a take-home part. The take home part will be handed out on Monday, April 4, during the test, and it will be due on Thursday, April 7 in class.*

Test	*Date*	*Time*
1 (Open Block)	Thursday, February 24,	12:00-1:20 *(Monday Schedule)*
2 (Open Block)	Monday, April 4,	12:00-1:20 *(and take-home part due Thursday, April 7 in class)*
Final	Thursday, May 12	12:00-2:00 p.m.

Note: Test 1 is the same time as a Math 38 test. If you are taking Math 38, please let me know well before the test, and I will arrange for an alternate time.

All complaints about the grading of an exam must be made in class on the day you get the exam back. Please indicate the problem you are questioning on the front of your blue book with the reason and give it to Todd. You must show your work in order to get credit for an answer on an exam (unless explicitly stated otherwise). Be sure to cross out work which you do not want to count.

You are required to sign your exam book. With your signature you are pledging that you have neither given nor received assistance on the exam. Students found violating this pledge will receive an F in the course and will be reported to the Dean of Students.

Makeups will be given only in extreme circumstances. Any conflicts or scheduling problems should be discussed well before the test and as soon as you know. Any changes to this syllabus will be announced in class.

Grading: Your grade will be figured using the following cool formula:

$$\begin{aligned}\text{grade} &= \max\{[15\%(HW + \text{class participation}) + 25\%T1 + 25\%T2 + 35\%F],\\ &\qquad [11\%(HW + \text{class participation}) + 22\%T1 + 22\%T2 + 45\%F]\}\end{aligned}$$

Note that homework and class participation will count either 15% or 11% depending on how they compare to your test grades. In either case, they count substantially more than in elementary courses.

Important Dates. You may add courses up to February 3. February 17 is the last day to take a course *pass-fail* and for sophomores, juniors, and seniors to drop without a **W**. First-year students may drop without a **W** until April 7. Finally, all students may drop with a **W** up until the last day of classes, May 2.

Math 136 Syllabus

Here is a list of what we will cover each week. This is subject to change; in particular, we might not cover everything listed in brackets. In any case, we will proceed through the syllabus in order until classes end. Changes to this syllabus will be announced in class.

Dates.	**Topics (sections in book).**
January 20, 25	Definition of derivative (6.1). Derivative as a linear operator (matrix) (6.2).
January 27	Basic theorems and examples (6.2, 6.3). Conditions for differentiability (6.4),
February 1	Chain Rule (6.5), [Product Rule 6.6], Mean value Theorem (6.7).
February 3, 8	Inverse and Implicit Function Theorems (7.1-3).
February 10	Summary of differentiation Integration in $\mathbb{R}$.
February 15, 17	Basic ideas for integration in $\mathbb{R}^n$ (8.1).
February 22	Characteristic functions (8.2) and review

Test 1, Thursday, February 24, 12:00-1:20. *(Monday schedule)*

March 1, 3	Sets of measure zero (8.2). Lebesgue's Theorem (8.3).
March 8, 10	Lebesgue's Theorem and Corollaries (8.3). Properties of integrals (8.4).
March 15, 17	Improper integrals on finite intervals in $\mathbb{R}$ (8.5). Convergence theorems for integrals (8.6).

Happy Spring Break!

March 29, 31	Fubini's Theorem (9.1, 9.2), Summary of integration, review.

Test 2, Monday, April 4, 12:00-1:20. Take home part handed out today. Take home part due April 7 in class

April 5, 7	Inner product spaces (10.1), ℓ_2.
April 12, 14	Convergence in mean, Orthogonal families of functions (10.2). Bessel's Inequality, Parseval's Theorem (10.2).
April 19, 21	Weierstrass M-Test, Completeness of the trigonometric system and convergence of Fourier series (10.3).
April 26, 28	[More convergence theorems (10.6)], [Applications of Fourier series (10.7)], review

Final Test, Thursday, May 12, 12:00-2:00 p.m.