Chapter 1: Overview of Heat Transfer

1. Solution:
   a. Radiation from the sun is absorbed by the upholstery.
   b. Heat from the electronics is transferred to the air by forced convection.
   c. Potatoes are heated by the hot water by convection. In addition to this, heat is transferred to the interior of the potato by conduction. The water is heated gets heat from the pot by convection, and the pan is heated by conduction from stove. The sides of the pot and the top surface of water will lose some heat to the surrounding air by convection.
   d. Heat from heating elements is transferred to the air inside the oven by convection. The heat is then transferred via natural or forced convection to the turkey. The interior of the turkey is heated by conduction. Heating coils and oven walls give off radiation which further heats the turkey.
   e. The heat from the room air is transferred to the ice cube and tray by natural convection. The metal tray transfers heat to the ice cube by conduction.

2. Given: A loaf bread at 120 ºC is cooling in air at 20 ºC. The dimension of the loaf is as 10 cm by 12 cm by 20 cm. h = 10 W/m^{2}K, e_{loaf} = 0.76, k_{loaf} = 0.121

   Find: Total heat loss from the bread.

   Assumptions: Conduction from the loaf to cooling rack is negligible due to the small area involved. The heat loss is therefore a combination of convection and radiation.

   Solution:
   Heat loss by convection is given by:
   $$q_{conv} = -hA_{loaf}(T_{loaf} - T_{¥}).$$

   Area of the loaf is:
   $$A_{loaf} = 2\,(0.2 \times 0.12) + 2\,(0.1 \times 0.12) + 2\,(0.2 \times 0.1)$$
   $$= 0.112 \text{ m}^2$$

   Convective heat loss is:
   $$q_{conv} = -hA(T_{loaf} - T_{¥})$$
   $$= -\,(10)(0.112)(120 - 20)$$
   $$= -112 \text{ W}$$

   Heat loss by radiation is:
   $$q_{rad} = -e_{loaf}\, s\, A_{loaf}T^4$$
   $$= -\,(0.76)(5.67 \times 10^{-8})(0.112)(120 + 273)^4$$ ← Note that T is in Kelvins.
   $$= -115.13 \text{ W}$$

   Total heat loss is:
   $$q_{total} = q_{rad} + q_{conv}$$
   $$= \mathbf{-\ 227.13\ W}$$ **← Solution**

3. Solution: Heat from the coffee is conducted through the cup. The outside surface of the cup loses heat to the surrounding air by convection. The bottom of the cup loses heat to the table by conduction. Natural convection also occurs at the top of the coffee. A very small amount of heat is also lost by radiation from sides of the cup and the top of coffee.

Chapter 2: Cooking Methods and Materials

1. Solution: Stainless steel is a durable, non-reactive material highly desired for cooking; however, it has one major drawback of having a relatively low thermal conductivity. A pot made of stainless steel may therefore suffer from poor temperature uniformity (which leads to other problems such as warping). A thick bottom helps distribute the heat more evenly. Furthermore, embedding material with high conductivity such as aluminum enhances the temperature uniformity even more. The thick bottom also helps in heat retention.

2. Solution: The tea poured into aluminum cup is likely to cool faster. Aluminum has a very high thermal conductivity and thermal diffusivity. The heat from the tea conducts through the cup, and then is lost to the surrounding by convection. Since aluminum has a higher thermal conductivity and diffusivity, the tea transfers its heat more readily to an aluminum cup than to the ceramic. Furthermore, due to the high thermal conductivity, the outside surface of the aluminum cup will be hotter than the outside surface of the ceramic cup. Since heat loss due to convection is proportional to ($T_{cup} - T_{air}$), a higher outside temperature promotes heat loss due to convection.

Chapter 3: Steady-State Conduction

1. Given: A rectangular slab measures 20 cm by 20 cm, by 2 cm thick.
T_1 = 100 ºC, T_2 = 96.2 ºC. k_{slab} = 170 W/mK.

Find: Rate of heat transfer through the slab.

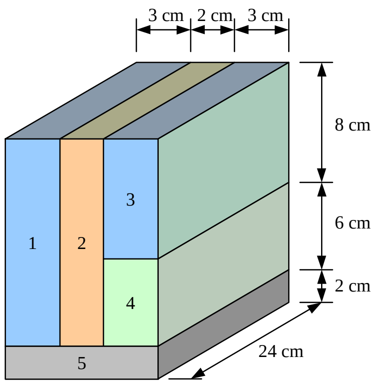
Assumptions: 1-D steady-state conduction.

Solution:
Heat transfer by conduction is given by:

$$q_{cond} = -kA\, DT/Dx$$
$$= -(170)(0.2^2)(100-96.2)/(0.02)$$
$$= (-)\mathbf{1292\ W}$$ ← **Solution**

Note: It is acceptable to drop the (–) sign in this case since the problem does not ask for the direction of heat transfer.

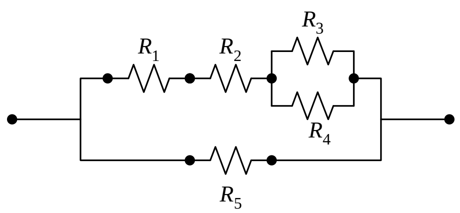
2. Given: A composite consisting of five sections, as shown in the figure. $k_1 = k_3$ = 80 W/mK, k_2 = 120 W/mK, k_4 = 100 W/mK, and k_5 = 150 W/mK.



Find: Construct the thermal circuit model and find the total thermal resistance.

Assumptions: 1-D steady-state conduction.

Solution:
The thermal circuit for the composite is:



To compute the total thermal resistance, first compute the thermal resistance for each component. For conduction, the thermal resistance is given by:

$$R_{cond} = L/kA$$
$$R_1 = 0.03 / (80 \times 0.0336)$$ ← $A_1 = 0.24 \times (0.08 + 0.06)$
$$= 0.0112$$

Similarly, R_2 = 0.005, R_3 = 0.0195, R_4 = 0.0208, and R_5 = 0.111.

The combined resistance for R_3 and R_4 in parallel is given by:

$$R_{34} = (R_3^{-1} + R_4^{-1})^{-1}$$

The combined resistance for R_1 through R_4 is:

$$R_{1234} = R_1 + R_2 + R_{34}$$

Finally, the total resistance for R_1 through R_5 is:

$$R_{total} = (R_{1234}^{-1} + R_5^{-1})^{-1}$$

$$= \left[\frac{1}{R_1 + R_2 + \left(\frac{1}{R_3} + \frac{1}{R_4} \right)^{-1}} + \frac{1}{R_5} \right]^{-1}$$

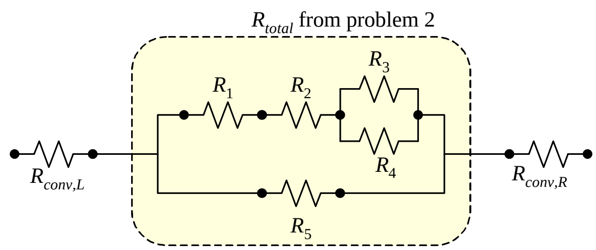
$= \mathbf{0.0212\ K/W}$ ← **Solution**

3. Given: Same composite as in problem 2. T_{left} = 20 ºC, T_{right} = 80 ºC; h = 15 W/m^{2}K on both sides.

   Find: Heat transfer rate through the composite.

   Assumptions: 1D steady-state conduction.

   Solution: To find the heat transfer rate, the thermal resistance from convection on both sides must also be included in series with the previously found R_{total}.



Thermal resistance from convection is given by:

$R_{conv} = 1 / hA$

The area is the total area of the slab:

$A = 0.24 \times (0.08 + 0.06 + 0.02)$
$= 0.0384\ m^2$

The thermal resistance due to convection on left side is:

$R_{conv,L} = 1 / (15 \times 0.0384)$
$= 1.736$

The thermal resistance due to convection on the right side is the same as that on left side. The net thermal resistance, including convection, is:

$R_{net} = 2R_{conv,L} + R_{total}$
$= 3.493$

The rate of heat transfer is given by:

$q = (T_{initial} - T_{final})/R_{total}$
$= (80 - 20)/3.493$
$= \mathbf{17.18\ W}$ ← **Solution**

Chapter 4: Transient Conduction

1. Given: Aluminum sphere with 2 cm diameter, in warm water bath.
   T_{water} = 40 ºC, $T_{initial}$ = 20 ºC. h = 400 W/m^{2}K.

   Find: Temperature at the center of the sphere at t = 20 sec.

   Assumptions: none.

   Solution: From Chapter 2, we find that k_{Al} = 170 W/mK, $c_{p\ Al}$ = 880 J/kgK, and r_{Al} = 2780 kg/m^3. Calculate the Biot number to determine whether the lumped body model is appropriate:

   Bi = hL_c/k
   = (400)(0.01/3) / 170 ← L_c for a sphere is r/3
   = 0.0078

   Since the criteria for using lumped body model is $Bi \leq 0.1$, the aluminum sphere can be treated as a lumped body. In this case, the temperature of the sphere at any given time is:

   $$\frac{\theta}{\theta_i} = \frac{T(t) - T_\infty}{T_i - T_\infty} = \exp\left(-\frac{hA}{mc_p}t\right)$$

   The mass of the sphere is calculated from the density:

   m = $r_{Al}\, 4p\, r^3/3$
   = (2780)(0.01^3) 4p/3
   = 0.0116 kg

   The area of the sphere is:

   A = $p\, r^2/4$
   = 7.85×10^{-5} m^2

   The normalized temperature of the sphere is:

   q/q_i = exp (-400 × 7.85E-5 × 10 / 0.0116 / 880)
   = exp (-0.031)
   = 0.9698

   The actual temperature of the sphere is:

   $T(t = 10)$ = $(q/q_i)(T_i - T_{¥}) + T_{¥}$
   = (0.9698)(20-40) + 40
   = 20.6 ºC **← Solution**

2. Given: A 4 cm potato in boiling water (T_{water} = 100 ºC). r_{potato} = 1050 kg/m3, $c_{p,potato}$ = 3.64 kJ/kgK, k_{potato} = 0.55 W/mK a_{potato} = 1.5×10^{-7} m^2/s. h = 400 W/m^{2}K

Find: temperature at the center and at r = 1 cm after 20 minutes.

Assumptions: none.

Solution: First, we check the Biot number to test for validity of lumped body assumption.

Bi = hL_C/k
= 400 × (0.02/2) / 0.55
= 7.3 > 0.1

So we cannot use the lumped body approximation. We will use the chart for sphere (Figure 4A.5) to solve this problem.

Bi^{-1} = k/hr_0
= 0.55 / (400×0.02)
= 0.07

Fo = at/r_0^2
= $(1.5\times10^{-7})(1200)/(0.02^2)$
= 0.45

From the chart for spheres, we read off q_0* ~ 0.04

T_0 = $q_0^*(T_i - T_{¥}) + T_\infty$
= 0.04 (20 – 100) + 100
= **96.8 °C** **← Centerline Temperature**

To find the temperature at r = 1cm, we use the second chart for spheres (Figure 4A.6).

r/r_0 = 0.5

Using Bi^{-1} found earlier, we read off q/q_0 ~ 0.65.

T = $(q/q_0)(T_0 - T_\infty) + T_\infty$
= 0.65 (96.8 – 100) + 100
= **97.92 °C** **← Temperature at r = 1cm**