

Homework 2 Solutions

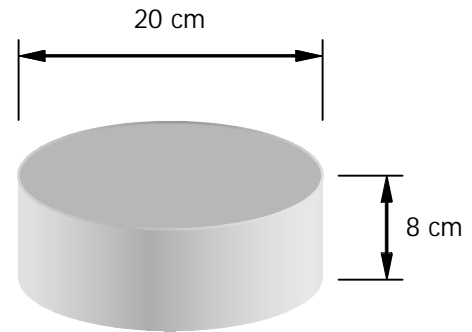
1. For each of the cases below, identify all modes of heat transfer (conduction, convection, or radiation) that contribute to the cooling or heating, and describe each. Clearly state the object(s) involved in each mode and the direction of heat transfer.

Solution: Appropriate responses for these problems include (but are not limited to):

- a) A slice of bread is heated in a toaster.
- Heat transfers from heating element in the toaster to the air by convection and radiation.
 - Heated air around the coil transfers heat to the toast by convection.
 - Some of the heat from the toast may be lost through convection. If the air inside the toaster hasn't warmed up yet.
 - Heat transfers from heating element to toast by convection.
 - In a pop-up type toaster, there is direct contact between the heating wire and the toast, so heat travels by conduction from heating wire to the toast as well.
- b) A kettle is filled with cold water and is heated on an electric stovetop until the water boils.
- Heat from the stove heating element travels to the bottom of the kettle by conduction (where there is direct contact) and radiation (where there is no contact).
 - Heat travels through the bottom surface of the kettle by conduction, and is transferred to the water by convection.
 - Water loses some heat to the side wall and top by convection, and the heat travels through the kettle top and sides by conduction.
 - There is convection all around the kettle – kettle loses heat to the air on the sides and top. It may gain or lose heat on the bottom side.
- c) A can of soda is placed in a refrigerator to cool.
- Heat from the soda travels to the can wall by convection.
 - Heat travels through the can by conduction, and is transferred to the refrigerator air by convection.
 - Can loses heat through the bottom by conduction (where the can is touching the fridge rack).
- d) Cake batter is poured into a cake pan, which is placed into an oven to bake.
- Heat travels from the heating element of the oven to air by convection.
 - The heated air transfers heat to the cake pan and the top surface of the cake by convection.
 - Heat travels through the cake pan by conduction.
 - Cake batter gets heat from oven air (top surface) and hot pan (side and bottom) by conduction (if you assume that the batter is not flowing – the cake is a solid) or convection (if you assume that the batter is fluid).
 - The heat travels through cake by conduction or convection (depends on whether you assume the cake to be solid or liquid).
 - Some amount of heat is transferred from the oven heating element to the pan by radiation.

2. After baking a cake in the oven, you set the cake on a rack to cool. The cake is 8 cm high, and has a 20 cm diameter. If the surface temperature of the cake is 90 °C on all sides and the air temperature in the room is 22 °C, calculate the rate of heat loss from the cake to the air. Some additional information are provided below.

- The convection coefficient can be approximated as 15 W/m²K on the top and the side surfaces, and 5 W/m²K on the bottom surface.
- The emissivity of the cake surface is 0.4 on all sides.
- You may neglect the effects of heat loss due to conduction.



Solution:

The cake is losing heat by convection on top, bottom, and side surfaces, and by radiation on all surfaces. The total heat loss is equal to the sum of all heat loss components:

$$q_{lost} = q_{convection, top} + q_{convection, top} + q_{convection, bottom} + q_{radiation, all sides}$$

where each term is given by:

$$q_{convection, top} = h_{top} A_{top} (T - T_{\infty})$$

$$q_{convection, bottom} = h_{bottom} A_{bottom} (T - T_{\infty})$$

$$q_{convection, side} = h_{side} A_{side} (T - T_{\infty})$$

$$q_{radiation, all sides} = \epsilon \sigma A_{total} T^4$$

We will first calculate all areas:

$$A_{top} = A_{bottom} = \pi r^2 = \pi(0.1)^2 = 0.03142 \text{ m}^2$$

$$A_{side} = 2\pi r h = 2\pi(0.1)(0.08) = 0.05027 \text{ m}^2$$

$$A_{total} = 2A_{top} + A_{side} = 0.1131 \text{ m}^2$$

Now calculate each component of heat loss:

$$q_{convection, top} = h_{top} A_{top} (T - T_{\infty}) = 15 (0.03142)(90 - 22) = 32.048$$

$$q_{convection, bottom} = h_{bottom} A_{bottom} (T - T_{\infty}) = 5 (0.03142)(90 - 22) = 10.683$$

$$q_{convection, side} = h_{side} A_{side} (T - T_{\infty}) = 15 (0.05027)(90 - 22) = 51.271$$

$$q_{radiation, all sides} = \epsilon \sigma A_{total} T^4 = 0.4(5.67 \times 10^{-8})(0.1131)(90+273)^4 = 44.54$$

Total heat loss is the sum of all components:

$$\begin{aligned} q_{lost} &= q_{convection, top} + q_{convection, top} + q_{convection, bottom} + q_{radiation, all sides} \\ &= 32.048 + 10.683 + 51.271 + 44.54 \\ &= \mathbf{138.5 \text{ W}} \end{aligned}$$