Homework 3 Solutions

1. A chef needs to purchase a new frying pan for use in a kitchen. The chef requires the pan to have the following qualities: 1) even heating, 2) good heat retention, 3) durable and low-maintenance surface. For each of the pans listed below, comment on their performance in the three requirement areas. Which pan(s) would you recommend to the chef?

   Solution: Appropriate responses for these problems include (but are not limited to):

   a) solid aluminum pan with anodized surface
      - aluminum has a very high thermal conductivity, so it promotes even heating.
      - heat retention of aluminum pans may be good if the pan has a thick construction (which is often the case with aluminum).
      - anodized surface has exellent durability and is scratch resistant. It is also low maintenance, requiring no seasoning or special cleaning.
      - this pan meets all three criteria (if it has thick construction), so it is recommended.

   b) solid aluminum pan with non-stick coating
      - this pan will heat evenly due to the high conductivity of aluminum.
      - as with (a), the pan will retain heat adequately if it has thick walls.
      - although non-stick coating will be low maintenance at first, it is not very durable and is prone to scratching.
      - I do not recommend this pan due to poor durability.

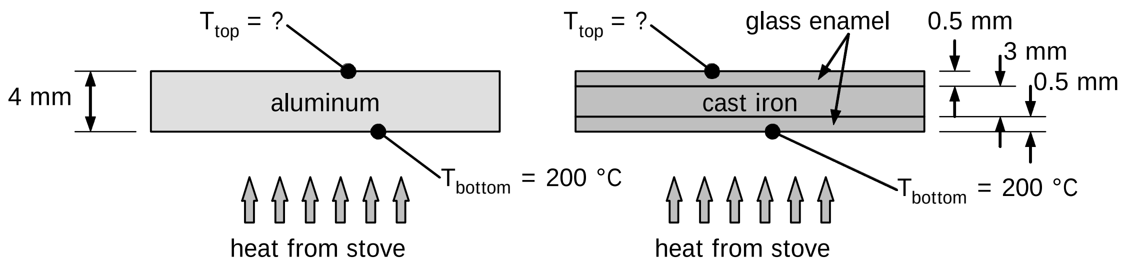
   c) solid cast iron pan
      - cast iron has a low thermal conductivity. However, it is typically made with very thick walls, which tends to even out the heat distribution.
      - cast iron has exellent heat retention, owing to its low thermal diffusivity and thick construction.
      - cast iron pans, if not regularly used, requires occasional seasoning to prevent rust.
      - I do not recommend this pan because of the need for regular maintenance (seasoning).

   d) stainless steel pan with aluminum core
      - stainless steel is a poor heat conductor; however, aluminum core should provide even heating for this pan.
      - stainless steel retains heat well, so this pan should be a better heat retainer than a solid aluminum pan of comparable shape.
      - stainless steel is a very durable surface, both scratch resistant and chemically non-reactive. It requires no special maintenance.
      - I recommend this pan because it meets all requirements.

   e) copper with stainless steel interior
      - copper is an excellent heat conductor, so the pan will have even heating.
      - this pan is not very likely to retain heat well, since copper is a poor heat retainer, and the stainless steel layer on the interior is probably not very thick.
      - copper is a soft metal, so the exterior of the pan may be prone to scratching. The interior, which is lined with stainless steel, will be durable. Copper pans require regular polishing to restore its surface quality.
      - I do not recommend this pan due to its marginal durability and need for regular polishing.

2. Given: a hot plate that can maintain a constant surface temperature of 200 °C. Two identically-sized skillets – one made of solid aluminum, and another made from cast iron with glass enamel coating – are placed on top of the hot plate, and heated until it reaches steady state.

- the pan has a circular bottom with 20 cm diameter
- aluminum pan is 4 mm thick
- cast iron pan has 3 mm thick cast iron core, surrounded on both sides by 0.5 mm of glass enamel coating (k_{enamel} = 2 W/mK)
- the top surface of the skillet is exposed to air at 20 °C, and convection coefficient on the top surface is 40 W/m²K
- the thermal conductivities are: k_{Al} = 170 W/mK, k_{iron} = 70 W/mK (from Ch.2).

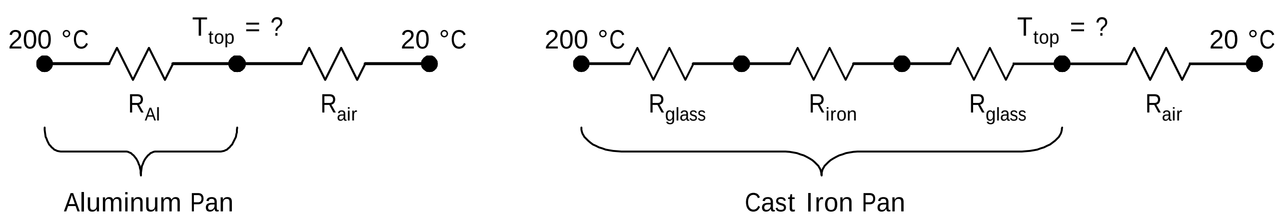


Find:

a) the amount of heat flow through each of the skillets; and b) the surface temperature at the top surface of each skillet.

Solution:

We will use the thermal circuit model for this problem. The pans can be represented by a thermal circuit as shown below:



The area is the same for all layers, and is $\pi r^2 = \pi(0.1)^2 = 0.03142$ m².

a) We will first find all thermal resistances.

Aluminum Pan:

$$R_{Al} = \frac{L_{Al}}{k_{Al}A} = \frac{0.004}{170(0.03142)} = 0.000749$$

Cast Iron Pan:

$$R_{iron} = \frac{L_{iron}}{k_{iron}A} = \frac{0.003}{70(0.03142)} = 0.00136\ ,\ R_{enamel} = \frac{L_{enamel}}{k_{enamel}A} = \frac{0.0005}{2(0.03142)} = 0.00796$$

Thermal resistance for convection:

$$R_{air} = \frac{1}{hA} = \frac{1}{40(0.03142)} = 0.7957$$

The total resistance from bottom of Al pan to air is:

$$R_{total,Al} = R_{Al} + R_{air} = 0.000749 + 0.7957 = 0.7964$$

The total resistance from bottom of glass enamel to air for cast iron pan is:

$$R_{total,iron} = R_{glass} + R_{iron} + R_{enamel} + R_{air} = 2(0.00796) + 0.00136 + 0.7957 = 0.8130$$

The heat flow is given by:

$$q_{Al} = \frac{\Delta T}{R_{total,Al}} = \frac{200 - 20}{0.7964} = \mathbf{226.0\ W} \text{ for Al pan}$$

$$q_{iron} = \frac{\Delta T}{R_{total,iron}} = \frac{200 - 20}{0.8130} = \mathbf{221.4\ W} \text{ for cast iron pan}$$

b) To find the temperature at the top surface, we look at the thermal circuit from the air to the top surface:

$$q = \frac{T_{top} - T_{air}}{R_{conv}}$$

Rearranging and substituting gives us:

$$T_{top,Al} = q_{Al}R_{conv} + T_{air} = 226.0(0.7957) + 20 = \mathbf{199.83\ ^\circ C} \text{ for Al pan}$$

$$T_{top,iron} = q_{iron}R_{conv} + T_{air} = 221.4(0.7957) + 20 = \mathbf{196.2\ ^\circ C} \text{ for cast iron pan}$$

Alternatively, you can look at the thermal circuit from the bottom of the pan to the top of the pan.