# Homework 4 Solutions

Given: A bottle of water cooling in a 2 °C refrigerator. Initial temperature of water is 20 °C.

$h = 1.2$ W/m²K, $k_{water} = 0.56$ W/mK, $\alpha_{water} = 15\times10^{-6}$ m²/s, $\rho_{water} = 1000$ kg/m³, $d_{water} = 8$ cm.

Find: the time for center of water to reach 6 °C using:

a) lumped body method (water is 30 cm high);

b) 1-D conduction model assuming bottle is infinitely long.

Solution:

a) For lumped body method:

First, we'll check to see that the Biot number is within acceptable range to use the lumped body method:

$$Bi = hL_c/k$$

The characteristic length ($L_c$) for a cylinder is r/2.

$$Bi = 1.2(0.04/2)/0.56$$
$$= 0.043$$

Since Bi < 0.1, we can proceed with the lumped body method.

The expression for temperature change using lumped body method is Eqn. 4.2 in the Lecture Notes:

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left(\frac{-hAt}{mc_p}\right)$$

To solve for time, we take the natural log of both sides and rearrange:

$$\ln\left(\frac{T - T_\infty}{T_i - T_\infty}\right) = \frac{-hAt}{mc_p}$$

$$t = -\frac{mc_p \ln\left(T - T_\infty / T_i - T_\infty\right)}{hA}$$

The surface area (A) of a cylinder with radius r and length L is:

$$A = 2\pi rL + 2\pi r^2 = 2\pi(0.04\times0.3 + 0.04^2) = 0.0855 \text{ m}^2$$

The mass (m) of the cylinder is [density × volume]:

$$m = \rho(\pi r^2 L) = 1000\pi(0.04^2\times0.3) = 1.51 \text{ kg}$$

The specific heat ($c_p$) is found from Eqn. 2.1 in Lecture Notes:

$$\alpha = k/\rho c_p \rightarrow c_p = k/\rho\alpha = 0.56/(1000\times15\times10^{-6}) = 37.33 \text{ J/kgK}$$

We now return to our expression for time:

$$t = -\frac{(1.51)(37.33)\ln[(6-2)/(20-2)]}{1.2(0.0855)} = 826 \text{ sec} = \mathbf{13.8\ minutes.}$$

b) Using the charts:

To use the charts, we first identify the "unknown." Since we're solving for *time* in this problem, our "unknown" in the chart is the Fourier Number (Fo), or the x-axis.

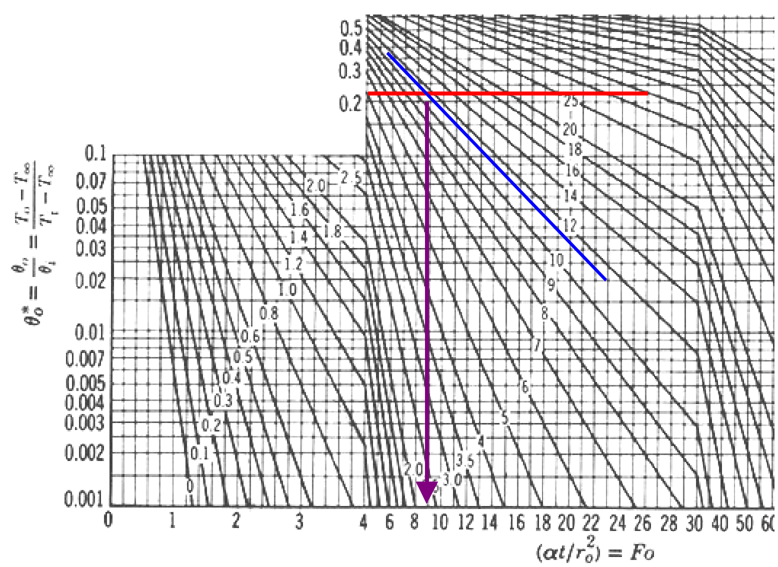
Referring to Figure 4A.3 in the Lecture Notes, we'll solve for $\theta_0$* (y-axis) and $Bi^{-1}$:

$$\theta_0^* = (T_0 - T_\infty)/(T_{init} - T_\infty)$$
$$= (6 - 2)/(20 - 2)$$
$$= 0.222$$

$$Bi^{-1} = k/hr_0$$
$$= 0.56/(1.2\times0.04)$$
$$= 11.67$$

Estimating $\theta_0$*= 0.222 and $Bi^{-1}$ = 11.67 on the graph (red and blue lines, respectively, on the figure below), we locate their intersection and draw a vertical line (purple line) to locate Fo ~ 9.

$$Fo = \alpha t/r_0^2$$
$$t = Fo\, r_0^2/\alpha$$
$$= 9(0.04^2)/15\times10^{-6}$$
$$= 960 \text{ sec}$$
$$= \mathbf{16\ min}$$




Partial section of Figure 4A.3 (centerline temperature for an infinite cylinder)