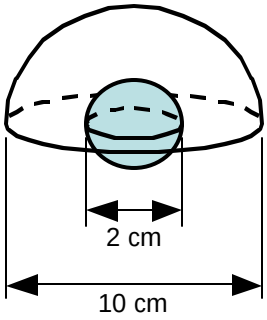
Homework 6 Solutions

Given:

2 cm diameter glass sphere at 100 °C, suspended in the air

10 cm diameter dome at 250 °C, with absorptivity and emissivity of 1, placed over the sphere so that the dome and sphere are concentric



The properties of sphere are:

$k = 1.4$ W/mK, $\rho = 2225$ kg/m³, $c_p = 835$ J/kgK, $\varepsilon = 0.9$, $\alpha = 0.8$

Find:

a) the rate of heat loss due to convection from the sphere if convection coefficient is 15 W/m²K and the air temperature is 20 °C.

b) the rate of heat loss from the sphere due to radiation.

c) the shape factor from the sphere to the dome.

d) the shape factor from the dome to the sphere.

e) the rate of heat loss from the inside surface of the dome.

f) the rate of heat transfer from the inside surface of the dome to the sphere.

g) the net heat transfer of the sphere. Is the sphere losing heat or gaining heat?

Solution:

a) The rate of heat loss due to convection is:

$$q_{conv} = -hA_{sphere}(T_{sphere} - T_{air})$$
$$= -(15)(4\pi 0.01^2)(100 - 20) = -1.50796 \text{ W, or } \mathbf{1.50796\ W\ of\ heat\ loss.}$$

b) The rate of radiative heat loss from the sphere is:

$$q_{rad,sphere} = -\varepsilon_{sphere}\sigma A_{sphere} T_{sphere}^4$$
$$= -(0.9)(5.67\times10^{-8})(4\pi 0.01^2)(100+273)^4 \times$$
$$= -1.24128 \text{ W, or } \mathbf{1.24128\ W\ of\ heat\ loss.}$$

c) The shape factor from the sphere to the dome is **50 %** or 0.5, because half of the radiation that leaves the sphere reaches the dome.

d) The shape factor from the dome to the sphere is found through reciprocity:

$$F_{dome\to sphere} = \frac{A_{sphere}}{A_{dome}} F_{sphere\to dome}$$
$$= \frac{4\pi r_{sphere}^2}{2\pi r_{dome}^2}(0.5) = \frac{2\left(0.01^2\right)}{0.05^2}(0.5)$$
$$= \mathbf{0.04,\ or\ 4\ \%}$$

e) The rate of heat loss from the inside surface of the dome is:

$$q_{rad,dome} = -\varepsilon_{dome}\sigma A_{dome} T_{dome}^4$$

$$= -(1)(5.67\times10^{-8})(2\pi 0.05^2)(250+273)^4 = \mathbf{-66.6361\ W.}$$

f) The rate of heat transfer from the dome to the sphere is the heat emitted by the dome (Part e), multiplied by the shape factor from dome to the sphere and the absorptivity of the sphere:

$$q_{dome\rightarrow sphere} = \alpha_{sphere} F_{dome\rightarrow sphere} \times [q_{rad,dome} \text{ from Part e}] = (0.8)(0.04)(66.6361) = \mathbf{2.1324\ W.}$$

Note: The sign is positive (+) from the point of view of the sphere, because this is heat gained by the sphere.

g) The net heat transfer of the sphere is:

$$q = q_{conv} + q_{rad,sphere} + q_{dome\rightarrow sphere} = -1.50796 + -1.24128 + 2.1324$$

$$= -0.6169 \text{ W, or } \mathbf{0.6169\ W \text{ of heat loss}}.$$

Note: The sign of the first two terms (q_{conv} and $q_{rad,sphere}$) is negative because they are heat loss from the sphere. The third term ($q_{dome\rightarrow sphere}$) is positive because it is heat gained by the sphere.