5-62 Heat Pump

Given: A heat pump is used to maintain a house at 23 °C. The house loses heat to the outside at a rate of 60,000 kJ/h, and the house generates 4000 kJ/h of heat. The COP of the heatpump is 2.5.

Find: Required power input to the house.

Solution:

The *COP* of a heat pump is defined as [what we want]/[what we put in]:

$$COP = \frac{Q_H}{W_{in}}$$

In this case, the heat pump needs to supply enough heat to the house to keep the house at constant temperature:

$$Q_H = Q_{lost} - Q_{generated} = 60{,}000 - 4000 = 56{,}000 \text{ kJ/h.}$$

$$W_{in} = \frac{Q_H}{COP} = \frac{56{,}000}{2.5} = 22{,}400 \text{ kJ/h} = \mathbf{6.22\ kW}.$$

5-81 Carnot Heat Engine

Given: A Carnot engine receives 650 kJ of heat from a source and rejects 200 kJ to a sink at 17 °C.

Find: a) Temperature of the source, and b) thermal efficiency of the heat engine.

Solution:

a) For a Carnot cycle, $Q_L = T_L\Delta S$ and $Q_H = T_H\Delta S$, and we can say that:

$\frac{T_H}{T_L} = \frac{Q_H}{Q_L}$, so the temperature of the source is $T_H = T_L \frac{Q_H}{Q_L} = 290\frac{650}{200}$ = **942.5 K**, or **669.5 °C**.

b) The thermal efficiency of a Carnot Engine is given by:

$$\eta_{th} = 1 - T_L/T_H = 1 - (290/942.5) = 0.692, \text{ or } \mathbf{69.2\ percent}.$$

5-96 Carnot Air Conditioner

Given: A reverse-Carnot cycle air conditioner transfers heat from a house at 750 kJ/min to maintain its temperature at 20 °C. The outside temperature is 35 °C.

Find: Power required to operate this air conditioner.

Solution:

The COP for a reverse-Carnot air conditioner is:

$$COP = \frac{1}{T_H/T_L - 1} = \frac{1}{308/293 - 1} = 19.53$$

This is also equal to [what we want]/[what we put in], or Q_L/W_{in}.

$$W_{in} = Q_L/COP = 750/19.53 = 38.40 \text{ kJ/min} = \mathbf{0.640\ kW}.$$

5-134 Carnot Heat Engine/Refrigerator

Given: A Carnot heat engine receives heat at 750 K and rejects heat to 300 K. The work output is used to drive a Carnot refrigerator that removes heat from a -15 °C space at a rate of 400 kJ/min and rejects the heat to 300 K environment.

Find: a) Rate of heat supplied to the heat engine, and b) total rate of heat rejection to the environment.

Solution:

a) The thermal efficiency of this Carnot heat engine is:

$$\eta_{th} = 1 - T_L/T_H = 1 - (300/750) = 0.6$$

For the refrigerator, the COP is:

$$COP = \frac{1}{T_H/T_L - 1} = \frac{1}{300/258 - 1} = 6.143$$

The work input into the refrigerator is:

$W_{in} = Q_L/COP = 400/6.143 = 65.12$ kJ/min → this is equal to W_{out} of the heat engine.

$Q_{in} = W_{out}/\eta_{th} = 65.12/0.6 =$ **108.53 kJ/min**, or **1.81 kW.**

b) The heat rejected into the environment by the Carnot Cycle is:

$Q_{out} = Q_L = Q_{in} - W_{in} = 108.53 - 65.12 = 43.41$ kJ/min.

The heat rejected by the refrigerator is:

$Q_{out} = Q_H = Q_L + W_{in} = 400 + 65.12 = 465.12$ kJ/min.

The total heat rejected is: 43.41 + 465.12 = **508.5 kJ/min**, or **8.47 kW**.

7-29 Exergy of R-134a

Given: Piston/cylinder device with 5 kg of R-134a at 0.8 MPa and 50 °C, cooled at constant pressure until it exists as liquid at 30 °C. Surroundings are at 100 kPa and 30 °C.

Find: a) exergy of the refrigerant at initial and final states, and b) exergy destroyed during this process.

Solution:

a) The exergy for closed systems, neglecting KE and PE, is:

$$X = m[u - u_0 + P_0(v - v_0) - T_0(s - s_0)]$$

At state 1, we have superheated vapor:
$v_1 = 0.02846$ m^3/kg, $u_1 = 261.62$ kJ/kg, $h_1 = 284.39$ kJ/kg, $s_1 = 0.9711$ kJ/kgK.

At state 2, we have saturated liquid at 30 °C:
$v_2 = 0.0008417$ m^3/kg, $u_2 = 90.84$ kJ/kg, $h_2 = 91.49$ kJ/kg, $s_2 = 0.3396$ kJ/kgK.

At the dead state, we have superheated vapor:
$v_0 = 0.24216$ m^3/kg, $u_0 = 254.54$ kJ/kg, $s_0 = 1.1122$ kJ/kgK.

$$X_1 = m[u_1 - u_0 + P_0(v_1 - v_0) - T_0(s_1 - s_0)]$$

$$= 5[261.62 - 254.54 + 100(0.02846 - 0.24216) - 303(0.9711 - 1.1122)] = \mathbf{142.3\ kJ}.$$

$$X_2 = m[u_2 - u_0 + P_0(v_2 - v_0) - T_0(s_2 - s_0)]$$

$$= 5[90.84 - 254.54 + 100(0.0008417 - 0.24216) - 303(0.3396 - 1.1122)] = \mathbf{231.3\ kJ}.$$

b) The exergy destroyed (irreversibility) is:

$$I = mT_0(s_2 - s_1) - Q.$$

From the 1st Law, $Q = m(h_2 - h_1) = 5(91.49 - 284.39) = -964.5$ kJ (heat loss).

$$I = 5(303)(0.3396 - 0.9711) - (-964.5) = \mathbf{7.8\ kJ}.$$

7-40 Exergy of air

Given: An insulated piston/cylinder device with 30 L of air at 120 kPa and 27 °C is heated for 5 min by a 50-W resistance heater in a constant pressure process. Th surroundings are at 100 kPa and 27 °C.

Find: The exergy destroyed during this process.

Solution:

Using constant specific heat ($c_p = 1.005$ kJ/kgK, $c_v = 0.718$ kJ/kgK, R = 0.287 kJ/kgK):

The mass of the air is: $m = P_1v_1/RT_1 = 120(30/1000)/(0.287\times300) = 0.0418$ kg.

Irreversibility is given by:

$$I = mT_0(s_2 - s_1) - Q = mT_0\left(c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}\right)$$; last term is zero because $P_1 = P_2$.

T_2 is found from the First Law:

$Q = mc_p(T_2 - T_1) \rightarrow T_2 = Q/mc_p + T_1 = (50\times5\times60/1000)/(0.0418\times1.005) + 300 = 657.1$ K.

$I = 0.0418(300)[1.005\ln(657.1/300)] =$ **9.88 kJ**

7-42 Irreversibility of Argon gas

Given: An insulated rigid tank divided into two equal parts by a partition. One part contains 3 kg of argon gas at 300 kPa and 70 °C, and the other part is evacuated. The partition is removed, and the gas fills the entire tank. The surroundings are at 25 °C.

Find: Exergy destroyed during this process.

Solution:

Using constant specific heat (c_p = 0.5203 kJ/kgK, c_v = 0.3122 kJ/kgK, R = 0.2081 kJ/kgK):

Since the tank is insulated, there is no heat loss during the process:

$Q = 0 = m\,c_v\,(T_2 - T_1) + W \rightarrow T_2 = T_1$ since $W = 0$.

Irreversibility is given by:

$$I = mT_0(s_2 - s_1) - Q = mT_0\left(c_v \ln\frac{T_2}{T_1} + R\ln\frac{V_2}{V_1}\right)$$; $Q = 0$ since the tank is insulated.

$$I = mT_0\left(R\ln\frac{V_2}{V_1}\right) = 3(298)\left(0.2081\ln\frac{2}{1}\right) = \mathbf{128.95\ kJ}.$$

7-67 Argon gas in compressor

Given: Argon gas enters an adiabatic compressor at 120 kPa and 30 °C with a velocity of 20 m/s and leaves at 1.2 MPa and 530 °C, with velocity of 80 m/s. The inlet area is 130 cm^2. The surroundings are at 25 °C.

Find: a) Reversible power input, and b) exergy destroyed.

Solution:

Using constant specific heat (c_p = 0.5203 kJ/kgK, c_v = 0.3122 kJ/kgK, R = 0.2081 kJ/kgK):

a) The reversible power input is the difference in exergy between exit and inlet:

$$\dot{W}_{rev} = \dot{X}_2 - \dot{X}_1$$

Exergy for Open Systems is:

$$\dot{X} = \dot{m}\left[h - h_0 + \frac{V^2}{2} - T_0(s - s_0)\right]$$, so $\dot{X}_2 - \dot{X}_1$ is:

$$\dot{W}_{rev} = \dot{m}\left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} - T_0(s_2 - s_1)\right]$$

$$= \dot{m}\left[c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2} - T_0\left(c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}\right)\right]$$

The mass flow rate is found from the inlet:

$v_1 = RT_1/P_1$ = 0.2081(303)/120 = 0.5255 m^3/kg

$$\dot{m} = \frac{V_1 A_1}{v_1} = \frac{20\left(130\,\frac{1}{10000}\right)}{0.5255} = 0.4948 \text{ kg/s}$$

$$\dot{W}_{rev} = 0.4948\left[0.5203(530 - 30) + \frac{80^2 - 20^2}{2000} - 298\left(0.5203\ln\frac{803}{303} - 0.2081\ln\frac{1200}{120}\right)\right] = \mathbf{126.14\ kW}$$

b) The exergy destroyed, or irreversibility, is:

$$\dot{I} = \dot{m}T_0(s_2 - s_1) = \dot{m}T_0\left(c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}\right)$$

$$= 0.4948(298)\left(0.5203\ln\frac{803}{303} - 0.2081\ln\frac{1200}{120}\right) = \mathbf{4.12\ kW}$$

7-71 Steam in nozzle

Given: Steam enters an adiabatic nozzle at 7 MPa and 500 °C with a velocity of 70 m/s, and exits at 5 MPa and 450 °C. The surroundings are at 25 °C.

Find: a) Exit velocity, b) isentropic efficiency, and c) exergy destroyed.

Solution:

a) The properties of steam at the entrance are: h_1 = 3410.3 kJ/kg, s_1 = 6.7975 kJ/kgK.

The properties at the exit are: h_2 = 3316.2 kJ/kg, s_2 = 6.8186 kJ/kgK.

The exit velocity is found from the First Law:

$$q = 0 = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}$$

$V_2^2 = 2(h_1 - h_2) + V_1^2 = 2000(3410.3 - 3316.2) + 70^2$ = 193,100, so V_2 = **439.4 m/s**.

b) To find the isentropic efficiency, we first need to find h_{2s}. At $s_{2s} = s_1$ and P_2, we are somewhere between 400 °C (s = 6.6459 kJ/kgK, h = 3195.7 kJ/kg) and 450 °C (s = 6.8186 kJ/kgK, h = 3316.2 kJ/kg). We interpolate to find h_{2s} = 3301.5 kJ/kg.

We can solve for V_{2s} using h_{2s}:

$V_{2s}^2 = 2(h_1 - h_{2s}) + V_1^2 = 2000(3410.3 - 3301.5) + 70^2$ = 222,500, and V_{2s} = 471.7 m/s.

The isentropic efficiency for a nozzle is:

$\eta = V_2^2/V_{2s}^2$ = 193,100 / 222,500 = 0.868, or **86.8 percent**.

c) The exergy destroyed, or irreversibility, per unit mass is:

$i = T_0(s_2 - s_1) = 298(6.8186 - 6.7975)$ = **6.288 kJ/kg**.

7-106 Two-stage steam turbine

Given: Steam enters an adiabatic, two-stage turbine at 8 MPa and 500 °C. It expands to 2 MPa and 350 °C in the first stage. The steam is then heated at constant pressure to 500 °C, then enters the second stage. At the exit of the second stage, the steam is at 30 kPa with quality of 97 percent. The total work output of the turbine is 5 MW, and the surroundings is at 25 °C.

Find: a) Reversible power output, and b) rate or exergy destroyed in the turbine.

Solution:

a) The properties at the entrance of first stage are: h_1 = 3398.3 kJ/kg, s_1 = 6.7240 kJ/kgK.

The properties at the exit of first stage are: h_2 = 3137.0 kJ/kg, s_2 = 6.9563 kJ/kgK.

The properties at the entrance of the second stage: h_3 = 3467.6 kJ/kg, s_3 = 7.4317 kJ/kgK.

A The properties at the exit of the second stage: h_f = 289.23 kJ/kg, h_{fg} = 2336.1 kJ/kg, s_f = 0.9439 kJ/kgK, s_{fg} = 6.8247 kJ/kgK.

$h_4 = h_f + x_4 h_{fg}$ = 289.23 + 0.97(2336.1) = 2555.2 kJ/kg

$s_4 = s_f + x_4 s_{fg}$ = 0.9439 + 0.97(6.8247) = 7.5639 kJ/kgK

The mass flow rate of the turbine is found from the First Law:

$$\dot{Q} = 0 = \dot{m}(h_2 - h_1) + \dot{m}(h_4 - h_3) + \dot{W}$$

$$\dot{m} = -\dot{W}/(h_2 - h_1 + h_4 - h_3) = -5000/(3137.0 - 3398.3 + 2555.2 - 3467.6) = 4.26 \text{ kg/s}$$

The reversible power output is given by $(\dot{X}_2 - \dot{X}_1) + (\dot{X}_4 - \dot{X}_3)$.

$$\dot{W}_{rev,1} = \dot{X}_2 - \dot{X}_1 = -\dot{m}[h_2 - h_1 - T_0(s_2 - s_1)]$$

$$= -4.26[3137.0 - 3398.3 - 298(6.9563 - 6.7240)] = 1407.6 \text{ kW}$$

$$\dot{W}_{rev,2} = -4.26[2555.2 - 3467.6 - 298(7.5639 - 7.4317)] = 4054.6 \text{ kW}$$

The total reversible work output is: 1407.6 + 4054.6 = **5462.3 kW**.

b) The total exergy destroyed is:

$$\dot{I} = \dot{m}T_0[(s_2 - s_1) + (s_4 - s_3)] = 4.26(298)(6.9563 - 6.7240 + 7.5639 - 7.4317) = \mathbf{462.7\ kW}.$$