8-36 Otto Cycle

Given: Otto cycle with compression ratio of 9.5. The air is at 100 kPa, 17 °C, and 600 cm³ prior to the compresion stroke. Temperature at the end of isentropic expansion is 800 K.

Find: a) Highest temperature and pressure in the cycle, b) amount of heat transferred, c) thermal efficiency, and d) mean effective pressure. Use constant specific heat.

Solution:

Properties of air at room temperature: c_p = 1.005 kJ/kgK, c_v = 0.718 kJ/kgK, k = 1.4, R = 0.287 kJ/kgK.

a) We given the P, T, and V before the compression stroke → these are P_1, T_1, and V_1.

Using the ideal gas law, we can find mass:

$m = P_1V_1/RT_1 = (100 \times 600\times10^{-6})/(0.287 \times 290) = 7.209\times10^{-4}$ kg

The temperature at the end of isentropic expansion is T_4. We also know that $V_4 = V_1 = V_{\max}$.

The highest temperature and pressure are at state 3. Since process 3→4 is isentropic, we can use the isentropic ratio between these states:

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{k-1} \rightarrow T_3 = T_4\, r^{\,k-1} = 800(9.5)^{1.4-1} = \mathbf{1968.7\ K.}$$

We know $V_{\min}$ from the compression ratio: $V_{\min} = V_3 = V_{\max}/r = 600/9.5 = 63.16$ cm³

We can find P_3 using the ideal gas law:

$P_3 = mRT_3/V_3 = (7.209\times10^{-4} \times 0.287 \times 1968.7)/(63.16\times10^{-6})$ = **6449 kPa**.

b) The heat transfer occurs between states 2 and 3. Using the First Law:

$Q_{in} = m\, c_v\, (T_3 - T_2)$.

We can find T_2 using the isentropic ratio between states 1 and 2:

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} \rightarrow T_2 = T_1\, r^{\,k-1} = 290(9.5)^{1.4-1} = 713.65\ \text{K.}$$

$Q_{in} = 7.209\times10^{-4} \times 0.718\,(1968.7 - 713.65)$ = **0.650 kJ**.

c) Thermal efficiency for an Otto cycle is given by:

$$\eta_{th} = 1 - \frac{1}{r^{k-1}} = 1 - \frac{1}{9.5^{1.4-1}} = 0.594, \text{ or } \mathbf{59.4\ percent}.$$

d) The mean effective pressure is defined as:

$$MEP = \frac{W_{net}}{V_{\max} - V_{\min}}$$

$W_{net} = Q_{in}\eta_{th} = 0.65 \times 0.594 = 0.3856$ kJ.

$$MEP = \frac{0.3856}{(600 - 63.16)\cdot 10^{-6}} = \mathbf{718.3\ kPa}.$$

8-45 Diesel Cycle

Given: Diesel cycle with a compresion ratio of 16 and a cutoff ratio of 2. The air is at 95 kPa and 27 °C at the beginning of compression process.

Find: a) temperature after heat addition, b) thermal efficiency, and c) mean effective pressure of the process. Use variable specific heats.

Solution:

a) We given the *P*, and *T* before the compression stroke → these are P_1 and T_1.

Properties at state 1 are: u_1 = 214.07 kJ/kg, P_{r1} = 1.3860, v_{r1} = 621.2

From the ideal gas law, we find $v_1 = RT_1/P_1$ = 0.287(300)/95 = 0.906 m^3/kg.

Process 1→2 is isentropic:

$$\frac{v_{r2}}{v_{r1}} = \frac{V_2}{V_1} = \frac{1}{r} \rightarrow v_{r2} = \frac{v_{r1}}{r} = \frac{621.2}{16} = 38.825.$$

We can interpolate between 860 K (v_r = 39.12, h = 888.27 kJ/kg) and 880 K (v_r = 36.61, h = 910.56 kJ/kg) to find h_2 = 890.89 kJ/kg and T_2 = 862.35 K.

We also know $v_2 = v_1/r$ = 0.876/16 = 0.05663 m^3/kg.

From the ideal gas law, we can find $P_2 = RT_2/v_2$ = 0.287(862.35)/0.05663 = 4370.8 kPa.

At state 3, we know that $P_3 = P_2$, and $v_3 = r_c\, v_2$ = 2(0.05663) = 0.11325 m^3/kg.

From the ideal gas law, we find $T_3 = P_3v_3/R$ = 4370.8(0.11325)/0.287 = **1724.7 K**.

We will interpolate between 1700 K (h = 1880.1 kJ/kg, v_r = 4.761) and 1750 K (h = 1941.6 kJ/kg, v_r = 4.328) to find h_3 = 1910.6 kJ/kg and v_{r3} = 4.546.

Process 3→4 is isentropic, so we can use the isentropic ratio to find state 4:

$$\frac{v_{r4}}{v_{r3}} = \frac{v_4}{v_3} \rightarrow v_{r4} = v_{r3}\frac{v_4}{v_3} = 4.546\,\frac{0.906}{0.11325} = 36.37$$

We can interpolate between 880 K (v_r = 36.61, u = 657.95 kJ/kg) and 900 K (v_r = 34.31, u_4 = 674.58 kJ/kg) to find u_4 = 659.69 kJ/kg.

b) The thermal efficiency for a Diesel cycle is:

$$h_{th} = 1 + \frac{u_1 - u_4}{h_3 - h_2} = 1 + \frac{214.07 - 659.69}{1910.6 - 890.89} = 0.563, \text{ or } \mathbf{56.3\ percent}.$$

c) The mean effective pressure is:

$$MEP = \frac{w_{net}}{v_{max} - v_{min}} = \frac{u_1 - u_4 + h_3 - h_2}{v_1 - v_2}$$

$$= \frac{214.07 - 659.69 + 1910.6 - 890.89}{0.906 - 0.05663} = \mathbf{675.9\ kPa}.$$

8-63 Stirling Cycle

Given: Stirling cycle with helium operates between temperature limits of 300 K and 2000 K and pressure limits of 150 kPa and 3 MPa. The mass of helium used in the cycle is 0.12 kg.

Find: a) thermal efficiency of the cycle; b) amount of heat transferred to the regenerator, and c) work output per cycle.

Solution:

Propertie of helium are: R = 2.0769 kJ/kgK, c_p = 5.1926 kJ/kgK, c_v = 3.1156 kJ/kgK, k = 1.667.

a) The thermal efficiency for a Stirling cycle is the same as the Carnot efficiency:

$\eta_{th} = 1 - T_L/T_H = 1 - (300/2000) = 0.85$, or **85 percent**.

b) The heat transferred to the regenerator is found by:

$Q_{\text{regen}} = -mc_v\ (T_3 - T_2) = mc_v\ (T_1 - T_4) = 0.12(3.1156)(2000 - 300) =$ **635.6 kJ**.

c) The work output per cycle is:

$$W_{net} = W_{1\to 2} + W_{3\to 4} = mRT_1 \ln\frac{V_2}{V_1} + mRT_3 \ln\frac{V_4}{V_3} = mR(T_H - T_L)\ln\frac{V_{\max}}{V_{\min}}$$

State 1 is at T_H and P_{max}.

From the ideal gas law, we can find $V_1 = mRT_1/P_1 = (0.12 \times 2.0769 \times 2000)/3000 = 0.16615\ \text{m}^3$.

State 3 is at T_L and $P_{\min}$, so using the ideal gas law we find V_3:

$V_3 = mRT_3/P_3 = (0.12 \times 2.0769 \times 300)/150 = 0.4985\ \text{m}^3$

$$W_{net} = 0.12 \cdot 2.0769(2000 - 300)\ln\frac{0.4985}{0.16615} = \mathbf{465.5\ kJ}.$$

8-73 Brayton Cycle

Given: Brayton cycle with air; pressure ratio is 12. Air enters the compressor at 300 K and enters the turbine at 1000 K. Net power output is 90 MW.

Find: The required mass flow rate for turbine and compressor isentropic efficiencies of a) 100 percent, and b) 80 percent. Use constant specific heats.

Solution:

Propertie of air are: R = 0.287 kJ/kgK, c_p = 1.005 kJ/kgK, c_v = 0.718 kJ/kgK, k = 1.4.

We are given T_1 = 300 K and T_3 = 1000 K.

a) For the ideal case, process 1→2 is isentropic, so we can use the isentropic ratio to find T_2:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} \rightarrow T_2 = T_1 (r_p)^{(k-1)/k} = 300(12)^{0.4/1.4} = 610.18 \text{ K}$$

We can do the same for process 3→4 to find T_4:

$$T_4 = T_3 \left(\frac{1}{r_p}\right)^{(k-1)/k} = 1000\left(\frac{1}{12}\right)^{0.4/1.4} = 491.66 \text{ K}$$

The net power output of the Brayton cycle is:

$$\dot{W}_{net} = \dot{m} c_p (T_1 - T_2 + T_3 - T_4)$$

$$\dot{m} = \frac{\dot{W}_{net}}{c_p (T_1 - T_2 + T_3 - T_4)} = \frac{90{,}000}{1.005(300 - 610.18 + 1000 - 491.66)} = \mathbf{451.9\ kg/s}.$$

b) The temperatures that we solved above are for the ideal case (T_{2s} , T_{4s}).

The actual work for the cycle is:

$$\dot{W}_{net} = \dot{m} c_p (T_1 - T_2)/\eta_C + \eta_T \dot{m} c_p (T_3 - T_4) = \dot{m} c_p \left[\frac{T_1 - T_2}{\eta_C} + \eta_T (T_3 - T_4)\right]$$

$$\dot{m} = \frac{\dot{W}_{net}}{c_p [(T_1 - T_2)/\eta_C + \eta_T (T_3 - T_4)]}$$

$$= \frac{90{,}000}{1.005[(300 - 610.18)/0.8 + 0.8(1000 - 491.66)]} = \mathbf{4726.5\ kg/s}.$$

8-79 Brayton Cycle

Given: Brayton cycle with air, with power output of 15 MW. The temperature limits are 310 K and 900 K, and the pressure ratio is 8. The compressor and turbine have isentropic efficiencies of 80 percent and 86 percent, respectively.

Find: The required mass flow rate for turbine using variable specific heat.

Solution:

We are given T_1 = 310 K → From the table, we can read off h_1 = 310.24 kJ/kg, P_{r1} = 1.5546.

Using the relative pressure ratio between 1 and 2, we get:

$$\frac{P_{r2}}{P_{r1}} = \frac{P_2}{P_1} \rightarrow P_{r2} = r_p P_{r1} = 8(1.5546) = 12.4368.$$

Interpolate between 550 K (P_r = 11.86, h = 555.74 kJ/kg) and 560 K (P_r = 12.66, h = 565.17 kJ/kg) to get h_{2s} = 562.54 kJ/kg.

The compressor work (per unit mass) is:

$$w_{c,actual} = \frac{w_{c,ideal}}{\eta_c} = \frac{h_1 - h_{2s}}{\eta_c} = \frac{310.24 - 562.54}{0.8} = -315.37 \text{ kJ/kg.}$$

At the turbine inlet, we have T_3 = 900 K → read off h_3 = 932.93 kJ/kg, P_{r3} = 75.29.

Using relative pressure ratios between 3 and 4, we get:

$$\frac{P_{r4}}{P_{r3}} = \frac{P_4}{P_3} = \frac{1}{r_p} \rightarrow P_{r4} = P_{r3}/r = 75.29/8 = 9.4113.$$

Interpolate between 510 K (P_r = 9.031, h = 513.32 kJ/kg) and 520 K (P_r = 9.684, h = 523.63 kJ/kg) to get h_{4s} = 519.32 kJ/kg.

The turbine work (per unit mass) is:

$$w_{t,actual} = \eta_{th}\, w_{t,ideal} = \eta_{th} (h_3 - h_{4s}) = 0.86(932.93 - 519.32) = 355.70 \text{ kJ/kg.}$$

The mass flow rate is:

$$\dot{m} = \frac{\dot{W}_{net}}{w_{c,actual} + w_{t,actual}} = \frac{15{,}000}{-315.37 + 355.70} = \mathbf{371.9\ kg/s}.$$

8-94 Brayton Cycle w/ Regeneration

Given: Regenerative gas-turbine engine. Air enters the compressor at 300 K and 100 kPa, and leaves the compressor at 800 kPa and 580 K. The regenerator effectiveness is 72 percent, and the air enters the turbine at 1200 K. The turbine has an efficiency of 86 percent.

Find: a) Amount of heat transfer in the regenerator, and b) thermal efficiency of the cycle. Use variable specific heats.

Solution:

At state 1, we have T_1 = 300 K and P_1 = 100 kPa. We can read off h_1 = 300.19 kJ/kg and P_{r1} = 1.3860.

At state 2, we have T_2 = 580 K and P_2 = 800 kPa. We read off h_2 = 586.04 kJ/kg and P_{r2} = 14.38.

The pressure ratio is $P_2/P_1 = 8$.

At the turbine inlet (state 3), we are at T_3 = 1200 K. h_3 = 1277.79 kJ/kg, P_{r3} = 238.0.

$$\frac{P_{r4}}{P_{r3}} = \frac{P_4}{P_3} = \frac{1}{r_p} \rightarrow P_{r4} = P_{r3}/r_p = 238/8 = 29.75.$$

This is between 700 K (P_r = 28.80, h = 713.27 kJ/kg) and 710 K (P_r = 30.38, h = 724.04 kJ/kg). Interpolate to find h_{4s} = 719.75 kJ/kg.

We can find h_{4a} using the turbine efficiency:

$$\eta = \frac{h_{4a} - h_3}{h_{4s} - h_3}$$

$\rightarrow h_{4a} = \eta(h_{4s} - h_3) + h_3 = 0.86(719.75 - 1277.79) + 1277.79 = 797.87$ kJ/kg.

The regenerator effectiveness is:

$$\varepsilon = \frac{h_5 - h_2}{h_4 - h_2}$$

$\rightarrow h_5 = \varepsilon(h_4 - h_2) + h_2 = 0.72(797.87 - 586.04) + 586.04 = 738.56$ kJ/kg.

a) The heat transfer in the regenerator is:

$q_{regen} = h_5 - h_2 = 738.56 - 586.04 =$ **152.5 kJ/kg**.

b) The thermal efficiency of the cycle is:

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{(h_1 - h_2) + (h_3 - h_4)}{h_3 - h_5} = \frac{300.19 - 586.04 + 1277.79 - 797.87}{1277.79 - 738.56} = 0.3599$$, or **36.0 percent**.

8-124 Exergy Destruction in Regenerative Brayton Cycle

Given: Regenerative Brayton cycle from problem 8-94. Sink is at 310 K, and source is at 1260 K, and P_0 = 100 kPa.

Find: a) Exergy destruction associated with each process, and b) exergy of the exhaust gases at the exit of the regenerator.

Solution:

a) To find the irreversibilities, we need to know h and s at each state. Taking information from problem 8-94, we get the following:

State 1: T_1 = 300 K, P_1 = 100 kPa, h_1 = 300.19 kJ/kg, $s_1°$ = 1.70203 kJ/kgK

State 2: T_2 = 580 K, P_2 = 800 kPa, h_2 = 586.04 kJ/kg, $s_2°$ = 2.37348 kJ/kgK

State 5: h_5 = 738.56 kJ/kg, $s_5°$ = 2.61289 kJ/kgK (interpolated between 720 K and 730 K)

State 3: T_3 = 1200 K, h_3 = 1277.79 kJ/kg, $s_3°$ = 3.17888 kJ/kgK

State 4a: h_{4a} = 797.87 kJ/kg, $s_{4a}°$ = 2.68733 kJ/kgK (interpolated between 760 K and 780 K)

State 6: using heat balance around the regenerator, we get:

$(h_6 - h_{4a}) = (h_2 - h_5) \rightarrow h_6 = h_2 - h_5 + h_{4a} = 586.04 - 738.56 + 797.87 = 645.35$ kJ/kg.

$s_6°$ = 2.46658 kJ/kgK (interpolated between 630 K and 640 K).

Irreversibility is: $i = T_0[(s_2 - s_1) - q/T]$

$i_{1\rightarrow 2} = T_0[s_2° - s_1° - R\ln(P_2/P_1)] = 310\,[2.37348 - 1.70203 - 0.287\ln(800/100)] =$ **23.14 kJ/kg**.

$i_{3\rightarrow 4} = T_0[s_4° - s_3° - R\ln(P_4/P_3)] = 310[2.68733 - 3.17888 - 0.287\ln(100/800)] =$ **32.63 kJ/kg**.

$i_{regen} = i_{2\rightarrow 5} + i_{4\rightarrow 6} = T_0(s_5° - s_2° + s_6° - s_4°)$
$= 310(2.61289 - 2.37348 + 2.46658 - 2.68733) =$ **5.78 kJ/kg**

$i_{5\rightarrow 3} = T_0[(s_3° - s_5°) - (h_3 - h_5)/T_{source}]$
$= 310[(3.17888 - 2.61289) - (1277.79 - 738.56)/1260] =$ **42.79 kJ/kg**

$i_{6\rightarrow 1} = T_0[(s_1° - s_6°) - (h_1 - h_6)/T_{sink}]$
$= 310(1.70203 - 2.46658) - (300.19 - 645.35) =$ **108.15 kJ/kg**

b) Exergy of exhaust gas is at state 6. At 310 K, h_0 = 310.24 kJ/kg and $s_0°$ = 1.73498 kJ/kgK.

$x_6 = h_6 - h_0 - T_0(s_6° - s_0°) = 645.35 - 310.24 - 310(2.46658 - 1.73498) =$ **108.3 kJ/kg**.

8-136 Diesel Cycle (using constant specific heat)

Given: Diesel cycle with air. The volume before and after the compression process are 1200 cm³ and 75 cm³, respectively, and volume after the heat addition process is 150 cm³. Air is at 17 °C and 100 kPa before the compression process.

Find: a) Pressure at the beginning of heat rejection process, b) net work per cycle, and c) mean effective pressure.

Solution:

Propertie of air are: R = 0.287 kJ/kgK, c_p = 1.005 kJ/kgK, c_v = 0.718 kJ/kgK, k = 1.4.

a) At state 1, we know that T_1 = 290 K, P_1 = 100 kPa, and V_1 = 1200 cm³.

We can find mass using the ideal gas law: $m = P_1V_1/RT_1$ = (100 × 1200×10^{-6}) / (0.287 × 290) = 0.001442 kg.

At state 2, we know V_2 = 75 cm³. Using isentropic ratio, we find T_2:

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} \rightarrow T_2 = T_1\left(\frac{V_1}{V_2}\right)^{k-1} = 290\left(\frac{1200}{75}\right)^{1.4-1} = 879.12 \text{ K}$$

From the ideal gas law, we find $P_2 = mRT_2/V_2$ = 0.001442×0.287×879.12/(75×10^{-6}) = 4850.3 kPa

At state 3, $P_3 = P_2$, and V_3 = 150 cm³. We find T_3 using the ideal gas law:

$T_3 = P_3V_3/mR$ = (4850.3 × 150×10^{-6})/(0.001442×0.287) = 1757.97 K.

At state 4, we know that $V_4 = V_1$. Using isentropic ratio from 3 to 4, we get:

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{k-1} = 1757.97\left(\frac{150}{1200}\right)^{1.4-1} = 765.2 \text{ K.}$$

P_4 can be found from the idea gas law:

$P_4 = mRT_4/V_4$ = 0.001442×0.287×765.2/(1200×10^{-6}) = **263.9 kPa**.

b) The net work is:

$W_{net} = m(u_1 - u_4 + h_3 - h_2) = m\,[c_v\,(T_1 - T_4) + c_p\,(T_3 - T_2)]$

= 0.001442[0.718(290 – 765.2) + 1.005(1757.97 – 879.12)] = **0.782 kJ**.

c) The mean effective pressure is:

$$MEP = \frac{W_{net}}{V_{max} - V_{min}} = \frac{0.782}{(1200 - 75)\cdot 10^{-6}} = \mathbf{695.1\ kPa}.$$

8-136 Diesel Cycle (using variable specific heat)

Given: Diesel cycle with air. The volume before and after the compression process are 1200 cm^3 and 75 cm^3, respectively, and volume after the heat addition process is 150 cm^3. Air is at 17 °C and 100 kPa before the compression process.

Find: a) Pressure at the beginning of heat rejection process, b) net work per cycle, and c) mean effective pressure.

Solution:

a) At state 1, we know that T_1 = 290 K, P_1 = 100 kPa, and V_1 = 1200 cm^3.

We can find mass using the ideal gas law:

$m = P_1V_1/RT_1 = (100 \times 1200\times10^{-6}) / (0.287 \times 290) = 0.001442$ kg.

We can also read off u_1 = 206.91 kJ/kg and v_{r1} = 676.1.

At state 2, we know V_2 = 75 cm^3. Using isentropic ratio, we find T_2:

$$\frac{v_{r2}}{v_{r1}} = \frac{V_2}{V_1} \rightarrow v_{r2} = v_{r1}\left(\frac{V_2}{V_1}\right) = 676.1\left(\frac{75}{1200}\right) = 42.256$$

Interpolating between 820 K and 840 K, we find h_2 = 863.01 kJ/kg and T_2 = 837.22 K.

From the ideal gas law, we find $P_2 = mRT_2/V_2 = 0.001442\times0.287\times837.22/(75\times10^{-6}) = 4619.85$ kPa

At state 3, $P_3 = P_2$, and V_3 = 150 cm^3. We find T_3 using the ideal gas law:

$T_3 = P_3V_3/mR = (4619.85 \times 150\times10^{-6})/(0.001442\times0.287) = 1674.45$ K.

Interpolate to find h_3 = 1848.69 kJ/kg and v_{r3} = 5.0039.

At state 4, we know that $V_4 = V_1$. Using isentropic ratio from 3 to 4, we get:

$$v_{r4} = v_{r3}\left(\frac{V_4}{V_3}\right) = 5.0039\left(\frac{1200}{150}\right) = 40.03.$$

Interpolate between 840 K and 860 K to find u_4 = 635.92 kJ/kg and T_4 = 853.33 K.

P_4 can be found from the idea gas law:

$P_4 = mRT_4/V_4 = 0.001442\times0.287\times853.33/(1200\times10^{-6})$ = **294.3 kPa**.

b) The net work is:

$W_{net} = m(u_1 - u_4 + h_3 - h_2) = 0.001442(206.91 - 635.92 + 1848.69 - 863.01)$ = **0.803 kJ**.

c) The mean effective pressure is:

$$MEP = \frac{W_{net}}{V_{\max} - V_{\min}} = \frac{0.803}{(1200-75)\cdot 10^{-6}} = \mathbf{713.5\ kPa}.$$