Homework 10

Due: Tuesday, December 1

1. Consider the system

$$\begin{aligned} \dot{x} &= -y - \alpha x \\ \dot{y} &= x - \alpha y \end{aligned}$$

If $\alpha \neq 0$, the solutions are spirals:

$$x(t) = re^{-\alpha t} \cos t \ , \quad y(t) = re^{-\alpha t} \sin t \ .$$

They spiral into the origin if $\alpha > 0$, and out of the origin if $\alpha < 0$. If $\alpha = 0$, the solutions are circles. Let's think about the idea of the Lorenz map for this very simple linear system. So let y_1, y_2, ... be the local maximum values of $y(t)$. If you plot the points (y_n, y_{n+1}) in a plane, what do you get? How can you distinguish, from that plot, the stable case ($\alpha > 0$) from the unstable one ($\alpha < 0$)?

2. Problems 10.1.12 and 10.1.13. (A fixed point x^* of f is "superstable" if $f'(x^*) = 0$.)

3. Problem 10.1.14. (You may or may not find Strogatz's hint useful.)

4. Problem 10.3.13a. (If you understand how a rainbow comes about, then also explain how those dark tracks are similar to rainbows.)