Homework 5

Due: Tuesday October 20 (Please note: Homework 6 will be due on Friday, October 23)

1. p. 141, problem 5.1.4.

2. p. 141, problem 5.1.7.

$*$ 3. p. 143, problem 5.2.13. The parameters m, b, and k are assumed positive.

In part a), also determine the trace τ and the determinant Δ, and show where in Fig. 5.2.8 we are here. (The answer depends on the parameters.)

In part b), if your answer is for instance "a stable node" (which is the correct answer for certain ranges of the parameter values), compute the eigenvalues and eigenvectors, and plot the phase portrait with reasonable care. This could become messy if you go about it in an awkward way, so follow these suggestions:

To compute the eigenvectors, first compute the eigenvalues. Then recall the system that an eigenvector $\underline{x}$ must solve: $A\underline{x} = \lambda\underline{x}$. Substitute for A the matrix you found in part a), but don't substitute the expressions for λ that you found yet. You get two equations for the components of $\underline{x}$. But when λ is an eigenvalue, then the first of these equations implies the second! (Why?) So you can disregard the second, and from the first, write down an eigenvector associated with the eigenvalue λ. Only now substitute the expressions for λ that you found.

To draw the phase portrait in the case when the origin is a stable node, ask yourself which is the fast eigendirection, and which is the slow one, and compare Fig. 5.2.3.

In part c), if you don't know those "standard notions", you'll find it easy to guess what they might mean once you have done parts a) and b)!

4. p. 144, problem 5.3.3. Assume $a > 0$ and $b > 0$. What is the interpretation of those assumptions? Show that you get a saddle. Compute the stable and unstable manifolds. The stable manifold is the border between heaven and hell here — explain why.

5. p. 144, problem 5.3.6. Asssume $a > 0$ and $b > 0$. That is, Juliet likes being loved, and she likes being in love. It's her very bad fortune that Romeo is a robot. Hint: Draw the set of all fixed points. There are more than just the origin here. Then draw typical trajectories.