Math 50, Fall 2009, Final Review Sheet

The final exam for Math 50 will take place on Friday, December 18, from 3:30 to 5:30 p.m. in Anderson 206. To prepare the exam, do the following problems thoroughly, and leave it there.

1. [linear approximations and Taylor approximations] (a) Approximate $\sqrt{99}$ using the linear approximation of $\sqrt{x}$ at $a = 100$. (b) Do the same using the quadratic approximation. (c) Do the same using the cubic approximation.

2. [Taylor approximations] Find the cubic Taylor approximation of $\tan x$ at $a = 0$.

3. [approximate differentiation] Explain, using Taylor expansions, why

$$\frac{u(x+h) - 2u(x) + u(x-h)}{h^2} \approx u''(x) + \frac{u''''(x)}{12}h^2$$

if u is four times differentiable.

4. [definition of the hyperbolic functions] page 259, problems 7–10.

5. [computing work using integrals] page 447, problem 29.

6. [computing volumes using integrals] pages 431/432, problesm 49, 50, 52.

7. [averages and integrals] The following temperatures were measured on a given day:

midnight	36^o
6 a.m.	36^o
8 a.m.	32^o
noon	30^o
4 p.m.	28^o
8 p.m.	26^o
midnight	22^o

Give a good approximation to the average temperature on that day. Explain your answer carefully. (Notice that the second measurement was taken at 6 a.m., not at 4 a.m.)

8. [techniques of integration] pages 518/519, problems 1, 2, 3, 4, 5, 7, 10, 15, 24, 41, 43, 47, 48, 49

9. [infinite series] page 759, problems 11–21. (For the convergent series, state whether they are conditionally or absolutely convergent.) Also problems 27, 30, 31.

10. [powers series and infinite Taylor expansions] page 760, problems 33, 40, 42, 45

11. [parametric curves] Sketch the curve given by $x(t) = t + 1$, $y(t) = t^2 - t$, $0 \leq t \leq 2$, and write down an integral representing its length. (Do not evaluate the integral.)

12. [differential equations] Find a function $y = y(t)$ that satisfies the equation

$$\frac{dy}{dt} = -y^2$$

together with the initial condition $y(1) = 1$. Explain every step, as on your homework problem about differential equations.

13. [fundamental theorem of Calculus] **This will be on the final exam:** State the fundamental theorem of calculus. Answer:

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$

This is the basic theorem, which I want you to know. It *implies* the fact that you all know:

$$\int_a^b f(x)dx = F(b) - F(a)$$

if $F'(x) = f(x)$.