Homework due on Tuesday, October 27

Trigonometric integrals: Section 7.2, problems 1, 12, 66.

Integrals involving inverse trigonometric and hyperbolic functions: Here are three facts that will be useful in doing the integration problems given below:

$$\begin{aligned} \frac{d}{dx}\arcsin(x) &= \frac{1}{\sqrt{1-x^2}} \\ \frac{d}{dx}\arctan(x) &= \frac{1}{1+x^2} \\ \frac{d}{dx}\operatorname{arcsinh}(x) &= \frac{1}{\sqrt{1+x^2}} \end{aligned}$$

Armed with these facts, compute the following integrals:

$$\text{a)} \int \frac{1}{\sqrt{4-x^2}}\,dx \qquad \text{b)} \int \frac{1}{3+4x^2}\,dx \qquad \text{c)} \int \frac{1}{\sqrt{3+4x^2}}\,dx$$

$$\text{d)} \int \frac{1}{\sqrt{x^2+4x+20}}\,dx \qquad \text{e)} \int \sqrt{1-x^2}\,dx$$

(To obtain credit, you must show how you reduce these integrals to the formulas that I gave you above. The correct answer will not be sufficient to get you credit.)

Hint for problem e): This involves three tricks. First, you do integration by parts, with $v = x$. Then, you have to realize that

$$\frac{x^2}{\sqrt{1-x^2}} = -\frac{1-x^2}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}} = -\sqrt{1-x^2} + \frac{1}{1-x^2}.$$

Finally, you also have to remember that not all is lost if you derive an equation that expresses an integral I that you wanted to compute in terms of I — you may be able to solve the equation for I.

PLEASE TURN PAGE

Integration by partial fraction decomposition: Use partial fraction decomposition to compute

$$\text{a)} \int \frac{1}{1-x^2}\,dx \qquad \text{b)} \int \frac{1}{x(1-x)}\,dx \qquad \text{c)} \int \frac{1}{x^2-5x+6}\,dx$$