Math 50, fall 2009, review for second midterm exam

The exam will take place on Friday, November 13, in our usual classroom. You will not be allowed to use notes, books, or calculators.

1. You are given the following information:

$$\arcsin'(x) = \frac{1}{\sqrt{1-x^2}}, \quad \arccos'(x) = -\frac{1}{\sqrt{1-x^2}}, \quad \arctan'(x) = \frac{1}{1+x^2}, \quad \int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}.$$

Armed with this information, calculate the following integrals:

$$(a)\ \int \frac{\cos x}{\sin^2 x}\,dx \qquad (b)\ \int x^2 e^{-x}\,dx \qquad (c)\ \int_0^{1/2} \frac{\arcsin(x)}{\sqrt{1-x^2}}\,dx$$

$$(d)\ \int \sqrt{t}\sin\left(1+t^{3/2}\right)\,dt \qquad (e)\ \int \sin^3 x\cos^2 x\,dx \qquad (f)\ \int \ln x\,dx$$

$$(g)\ \int \sin^2 u\cos^2 u\,du \qquad (h)\ \int_3^4 \frac{x}{x^2-3x+2}\,dx \qquad (i)\ \int \frac{1}{2x^2+3}\,dx$$

$$(j)\ \int_{-1}^{1} \frac{1}{\sqrt{|x|}}\,dx \qquad (k)\ \int_0^{\infty} xe^{-x}\,dx \qquad (l)\ \int_{-\infty}^{\infty} x^2 e^{-x^2}\,dx$$

2. Draw a picture showing that for any natural number $n \geq 1$,

$$\sum_{i=1}^{n} \frac{1}{\sqrt{i}} < \int_{1/2}^{n+1/2} \frac{1}{\sqrt{x}}\,dx = 2\sqrt{n+1/2} - 2\sqrt{1/2}.$$

3. Page 448, problem 5a.

4. Infinite series: page 722, all odd-numbered problems except problem 15

5. page 695, problem 58, parts (a) and (b). (The hint that the book gives is irrelevant for part (a), but relevant for part (b). The time required for the ball to *fall* is easy to compute. Each fall, except for the first fall from height H to the ground, is preceded by an *ascent*, and that ascent takes precisely as long as the fall following it — the upward motion is a mirror image of the following downward motion.)