problem 14 on page 441: Initially, the chain lies on the ground. At the end, a 6 m segment of the chain is raised up. The assumption here is that the raised segment is precisely vertical. (The problem would become very much harder if we assumed that the raise segment were curved, as in reality it typically would be.)

Think now about a small piece of the raised segment, of length dx, and assume that x denotes the height to which the lower edge of the small piece is lifted, so the upper edge is brought to height $x+dx$. The mass of this small piece is $8\,dx$, if mass is measured in kg, and height is measured in meters. (The chain has mass 8 kg per meter.) Its weight is therefore $8\,g\,dx$, where $g \approx 9.81\text{m/s}^2$ denotes the gravitational acceleration. When the raised segment was lifted, this small piece was lifted to height x, so the work done was $8\,g\,x\,dx$. Therefore the total work done to lift the raised segment was

$$\int_0^6 8gx\,dx = 8g\int_0^6 x\,dx = 144g \approx 1413.$$

The units here are kg m /s^2=J. A 60-Watt light bulb uses this much energy in about 24 seconds. ("60-Watt" means it uses 60 J per second.)

problem 17 on page 441: We can calculate the work needed to lift the bucket, and the work needed to pull up the rope, separately. First, think about just the rope, with no bucket attached. So think of standing on a roof, at a height of 12 m, and pulling up a rope, where initially the end of the rope is just touching the ground. Think about a short segment of the rope, of length dx, and initially at height x (in meters) above the ground. This segment much be raised by $12-x$ meters, and its weight is $0.8gdx$. So the work to raise it equals $(12-x)0.8gdx$, and the total work to raise the rope is

$$\int_0^{12}(12-x)0.8gdx = 0.8g\int_0^{12}(12-x)\,dx = 0.8g\times 72,$$

The units here are kg m/s^2=J.

Now we calculate the work needed to raise the bucket. At height x, the weight of the bucket, together with the water in it, equals $(10+36-3x)g$kg. (The "10" represents the weight of the bucket itself, and "$36-3x$" represents the weight of the water: It starts out at 36, and decreases linearly to 0 at $x=12$.) To lift the bucket from height x to height $x+dx$ therefore requires the work $(10+36-3x)gdx$J. To lift the bucket from height 0 to 12 requires, in J, the work

$$\int_0^{12}(10+36-3x)gdx = 336g.$$

So the total work is about

$$(0.8\times 72+336)g\text{ J} \approx 3,861\text{ J}.$$

(The book says 3,8571 J, but that's because they use $g \approx 9.8\text{m/s}^2$. I used the more accurate approximation $g \approx 9.81\text{m/s}^2$.)