TUFTS UNIVERSITY
Department of Economics

MathCamp Fall 2005

Solutions to practice prob I

1. (i) Differentiate w.r.t. π and solve the f.o.c for π.

(ii) Use the standard procedures for curve sketching: (i) find where curve intersect x– and y–axes; (ii) find turning points if any (max, min); (iii) find regions of increase, and decrease; (iv) find vertical and horizontal asymptotes, etc

2. Same as 1(ii).

3. Use the formula developed in class, which says the asset value is maximized at a point that its growth rate is equal to the rate of interest on the alternative investment: that is

$$\frac{dV/dt}{V} = r$$

at time T But $V = 2000e^{t^{0.25}}$. Take logs on both sides gives: $\ln V = ln2000 + t^{0.25}$. Then differentiate w.r.t t

$$\frac{dV/dt}{V} = 0.25t^{-0.75} = r$$

when $t = T$. Sub $r = 0.1$, and solve for T.

4. Try (i)

(ii) $z = 6x - x^2 + 16y - 4y^2$. There is only one critical point. $z_x = 0 \Rightarrow x = 3$; and $z_y = 0 \Rightarrow y = 2$. To find if max or min, you can use the Hessian matrix: $z_{xx} = -2, z_{xy} = z_{yx} = 0$ and $z_{yy} = -8$. So $det(H) = 16 > 0$. Therefore Hessian is negative definite at $(x, y) = (3, 2)$. Hence a (global) max.

5. Given, $Q = 3K^{2/3}L^{1/3}$, we know that (i) $Q = 1500$ when $K = 1000$ and $L = 125$.

(ii) The total change in Q due to small changes in K and/or L will be given by

$$dQ = \frac{\partial Q}{\partial K}dK + \frac{\partial Q}{\partial L}dL = 2\left(\frac{L}{K}\right)^{1/3} + \left(\frac{K}{L}\right)^{2/3}$$

Then Sub $K = 1000, L = 125$ and $dK = -2$, and $dL = 3$ and find dQ. The new output will be old plus change.

6. We know that $g(x) = f^{-1}(x)$; that is, g is the inverse of f. Re-write as: $f(g(x)) = x$, then differentiate w.r.t x. This gives: $f'(g(x))g'(x) = 1$.

Verify: Let $y = f(x) = \frac{x-1}{x+1}$. To find the inverse, solve the equation for x : this yields

$$x = \frac{1+y}{1-y}.$$

The inverse function can then be written as:

$$g(x) = \frac{1+x}{1-x}$$

(Note: y is just a dummy variable)

Differentiate g w.r.t. x gives: $g'(x) = \frac{2}{(1-x)^2}$. If we differentiate f w.r.t. x, we get $f'(x) = \frac{2}{(1+x)^2}$.

Finally, $f'(g(x)) = \frac{2}{(1+g(x))^2} == \frac{(1-x)^2}{2}$, using $g(x) = \frac{1+x}{1-x}$. So $g(x) = \frac{1}{f'(g(x))}$.

7. Do this.

8. (a) Check: $f(tx) = k(tx)^a = t^a(kx^a) = t^a f(x)$. Hence $f(x)$ is homogeneous of degree a.

(b) Totally differemtiate the two equations w.r.t b: This gives

$$\begin{aligned} F_x \frac{dx}{db} + F_y \frac{dy}{db} &= 0 \\ G_x \frac{dx}{db} + G_y \frac{dy}{db} &= 1 \end{aligned}$$

and then solve for $\frac{dx}{db}$ and $\frac{dy}{db}$.

9. We will do this in class.

10. First, use the second constraint to sub for L: i.e., $L = H - \ell$, where H is a fixed amount of time.

The we can write the Lagrangian as:

$$L(c, \ell, \mu) = \alpha \ln c + \beta \ln \ell - \mu(c - w(H - \ell))$$

where μ is the lagrange multilplier.

Then solve.