TUFTS UNIVERSITY
Medical School

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Course CMBA0264-01; Statistics with Applications; Summer 2009
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Assignment III – Answer Key

Data sets and other materials posted at the course web page:
http://www.tufts.edu/~mbiancon/CMBA0264-2009.html

Please, read carefully the material on Interval Estimation (adapted from Newbold, *et al*) available at the course web page to solve the problems below.

1. Consider the problem of estimating the mean yearly salaries of Northwest hospital administrators. Suppose the yearly salaries of administrators in the Northwest are $\mathcal{N}(\mu = ?, \sigma = 12)$ in thousands of dollars.

Suppose we take a sample of n=25 administrators and $\bar{\mathbf{X}} = 82$.
Give a 95% confidence interval for the population mean.

For our problem, $\sigma = 12$, n=25 and $\bar{x} = 82$. We are therefore, 95%

certain that $\{Z_{\alpha/2} = Z_{0.025}$ **= 1.96 (from table) or**

In STATA: . display invnorm(0.025)

-1.959964 }

$$82 - (1.96)\frac{12}{\sqrt{25}} < \mu < 82 + (1.96)\frac{12}{\sqrt{25}}$$ **which**

computes to

$$82-4.704 < \mu < 82 + 4.704 \equiv$$

$$77.296 < \mu < 86.704$$

Our final answer is: we are 95% certain 77,296 < μ < 86,704

2. Redo #1 for the case: $\mathcal{N}(\mu = ?, \sigma = ?)$; n=25, $\bar{\mathbf{X}} = 82$, s = 12.

Explain the differences if any.

In this case the variance is unknown. The sample size is small so we use the student's t distribution.

For this problem, s = 12 , n=25 and $\bar{x}$ = 82. For the 95% CI, $t_{n-1,\alpha/2} = t_{24,\ 0.025}$ **= 2.064 (from table) or**

In STATA:
. display invttail(24,0.025)
2.0638986

hence, 95% certain that 82 - (2.064) $\frac{12}{\sqrt{25}}$ **<** μ **< 82 + (2.064)** $\frac{12}{\sqrt{25}}$ **which**

computes to

82-4.9536 < μ **< 82 + 4.9536 ≡**

77.0464 < μ **< 86.9536**

Our final answer is: we are 95% certain 77,046 < μ **< 86,954**

hence, the interval is wider to the unknown variance and small sample size.

3. The amount of life insurance taken out by families earning $50,000 a year is normally distributed. Based on a sample of n=100, we find:

$\bar{x}$ **= 80,000 and s = 15,000. Wh**at will be a 99% confidence interval for μ = true mean amount of life insurance?

$$\bar{x} \pm t_{\nu,\alpha/2}\frac{s}{\sqrt{n}} \equiv \bar{x} \pm t_{99,.005}\frac{s}{\sqrt{100}}$$

where $t_{99,.005}$ **= 2.6264 is gotten from the t-table or STATA.**

If when we sample, collect our data and compute our statistics, the results are:

$\bar{x}$ **= 80,000 and s = 15,000 then we have**

$$80{,}000 - (2.6264)\frac{15{,}000}{10} < \mu < 80{,}000 + (2.6264)\frac{15{,}000}{10}$$

We are 99% confident $76{,}060.4 < \mu < 83{,}939.6$

4. Revisit the problem of estimating life insurance. You already found a confidence interval for the mean based on a sample of n=100 with the following statistics computed from the sample: $\bar{\mathbf{x}} = 80$ and s = 15 ($000). Find a 95% confidence interval for the population variance of life insurance.

If we want to find a confidence interval for the variance using this data, $\chi^2_{n-1,\alpha/2} = \chi^2_{99,.025}$ **=128.4219** $\chi^2_{n-1,1-\alpha/2} = \chi^2_{99,.975}$ **=73.6611**

In STATA:

```
. display invchi2tail(99,0.025)
128.42199
. display invchi2tail(99,0.975)
73.36108
```

Hence: $\frac{(n-1)s^2}{\chi^2_{n-1,\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{n-1,1-\alpha/2}}$ **=** $\frac{99(225)}{128.4219} < \sigma^2 < \frac{99(225)}{73.611}$

with 95% certainty $173.45 < \sigma^2 < 303.635$

5. Suppose you'd like to do a survey poll asking whether or not individuals like a consumer product. What is the needed sample size for a margin of error of at most 4%? Explain how you obtain your answer.

So, now we can find the sample size that determines a margin of error of 4% when doing a survey or poll as follows: $n = \frac{0.25(1.96)^2}{(.04)^2}$ =600.25. Answer is to take a sample of n=601.

Bianconi, July 2009