Math/Comp 128, Homework 2
Due Sept. 19, 2007

Instructions: Do 6 of the 8 problems

1. Text, 2.1

2. Show that the product of two orthogonal matrices is orthogonal.

3. Text, 2.2, part a only (the norm is the 2-norm).

4. Text, 3.1

5. (You may do this in Matlab, or use a calculator)

   Plot the following data points:

   $$a = (1, 3); b = (2, 1.7); c = (1.1, 2.9); d = (2.3, 1.8); e = (1.5, 2.6); f = (3, 2.5)$$

   For each data point, compute the expansion coefficients in the basis given by columns of the matrix

   $$\sqrt{2}/2 \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}.$$

   Give a geometric interpretation of why the 2nd expansion coefficients for points b,d,f are negative.

6. Implement the outer product version of the matrix-matrix product (see last assignment) in Matlab, and verify (by testing on some matrices where you know the answer a priori) that it works.

7. A *tridiagonal* matrix is one with its only non-zero elements residing on the main diagonal and first superdiagonal and first subdiagonal. Give an algorithm (you may use psuedo-code, or Matlab form) for computing the 1-norm of such a matrix in no more than $O(n)$ flops. Then, generalize your algorithm to banded matrices with bandwidth $2b + 1$ ($a_{i,j} = 0$, if $|i - j| > b$, for $i = 1, \ldots, n$ and $j = 1, \ldots, n$), and give the number of flops for your algorithm as a function of both n and b.

8. From my webpage, grab the Matlab .m file called dst.m. Use it to create an orthogonal matrix Z of size $n \times n$ for $n = 32$ by typing as follows: `Z = (sqrt(2/n+1))*dst(eye(n));` Now create a discrete approximation to $\cos(wx)$ for $x \in [0, \pi]$ as follows: `y=cos([0:n-1]'/n * w*pi)`, where w is an integer between 1 and n.

   (a) Compute and plot the expansion coefficients of y in the basis given by the columns of Z for several (at least 5) different values of w between 1 and n.

   (b) Based on your results above, what do you expect to see if you plot the expansion coefficients for `y = cos([0:n-1]'/n*3*pi) + cos([0:n-1]'/n*17*pi)` and why? (You can test it to see if you're right!) How about for `y=randn(n,1)`?

Math 250, Homework 2
Due Sept. 19, 2007

Instructions: Do all the following

1. Show the product of k orthogonal matrices is orthogonal

2. Text 2.2, both parts (the norm is the 2-norm)

3. Text 2.3

4. Text 3.1
5. Text 3.3a and c
6. Item 4 above
7. Either 3.4 OR 7 above OR 8 above