Math/ Comp 128
Homework 6
Due Oct. 31

Instructions. Do 5 of the 7 problems Note: Some problems may require you to implement Algorithm 21.1 in Matlab.

1. Suppose the $m \times m$ matrix A is partitioned into a 2×2 block form:

$$A = \begin{bmatrix} C & D \\ E & F \end{bmatrix},$$

where C is a $k \times k$ matrix that is invertible.

(a) Show that this matrix can be "block factored" into the form $A = LU$, where L and U are block matrices satisfying

$$A = \begin{bmatrix} I_{k\times k} & 0 \\ L_{2,2} & I_{m-k\times m-k} \end{bmatrix} \begin{bmatrix} U_{1,1} & U_{1,2} \\ 0 & U_{4,4} \end{bmatrix}.$$

In other words, determine the blocks $L_{2,2}, U_{1,1}, U_{1,2}, U_{4,4}$ as functions of the blocks of A so that this works. The subscripts on the identity matrix blocks indicate their sizes. Note that the matrix $U_{4,4}$ is called the *Schur complement*.

(b) Given the factorization you identified, give a step by step procedure (a few lines long) for using this factorization to solve the system

$$Ax = b.$$

Your procedure will depend on partitioning x and b accordingly into appropriate length subvectors. Note that although the Schur compliment block will have C^{-1} appearing in it, you should describe a general procedure that doesn't rely on computing C^{-1} explicitly.

2. Text, 21.5

3. Text, 21.3

4. Text, 22.2

5. Text, 23.1 (be sure to explain your reasoning)

6. Text, 23.2

7. Text, 23.3

Math 250

Instructions: Do 4 of the above, and the following 2 problems.

1. Text, 22.1
2. On page 176, the stability of the Cholesky decomposition is discussed. Give proofs of the two facts discussed here that are used in the proof of Theorem 23.2. That is
   - $\|R\|_2 = \|R^*\|_2 = \|A\|_2^{1/2}$
   - If A is Hermitian positive definite, pivoting is not needed because the largest entry must appear on the diagonal during the factorization process. (proof by induction)