Matlab Problems

Problem 1

(a) Create a vector representing a unit-amplitude sine wave of frequency 10 Hz over the period 0 to 10 seconds.

```
>> t=linspace(0,10,1000);
>> x=sin(2*pi*10*t);
```

Here I have used 1000 points between 0 and 10, which gives me enough points to generate a smooth curve (here I have 100 points per period).

(b) Compute the root-mean-square (RMS) value of the vector by using the commands sqrt(), mean(), and .^

```
>> sqrt(mean(x.^2))
```

The RMS of the wave should be the amplitude of the sine or cosine wave over sqrt(2).

(c) Now make the sine wave decay in time with time constant 3 seconds.

```
>> x=x.*exp(-t/3);
```

Compute the RMS value of this decaying wave from 0 to 10 seconds.

Problem 2

Solve the set of linear algebraic equations:

3a + 7b -1.5c = 9
3.2b + c = 2
(1/9)a -12b = 0

By solving a matrix equation of the form
Ax=b

Where here x is the vector [a;b;c].

(a) First, create the matrix A and the vector b.

```
>> A=[3 7 -1.5;0 3.2 1;1/9 -12 0]
>> b=[9;2;0]
```

(b) Then, use the backslash operator to solve for x. (type 'help slash' at the matlab prompt)

```
>> x=A\b;
```

Problem 3

Verify the trigonometric identity

$sin^2(t)+cos^2(t) = 1$

(a) First, create a time vector t, and two vectors $a=sin(t)$ and $b=cos(t)$.
(b) Then create a third vector which is the sum of the squares of a and b (REMEMBER to use the pointwise squaring operation .^).
(c) Plot the resulting vector verses time. It should equal 1 everywhere.

Problem 4

Image a train had acceleration

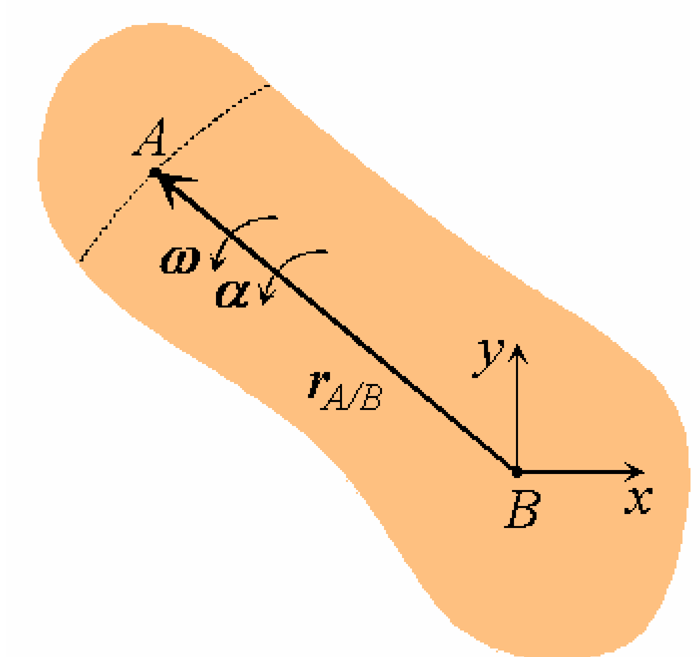
$a=2t$ where t is in seconds, a is in m/s

The velocity at t=2 s was 180 km/h.

(a) Create a time vector that goes from t=2 to t=4 seconds.
(b) Create an acceleration vector at those times using the above function (a=2t).
(c) Plot acceleration vs. time.
(d) Use the trapz command to numerically integrate the acceleration from 2 to 4 seconds to compute the change in velocity between 2 and 4 seconds. Add this to the initial velocity of 180 km/h (convert to m/s first!) given at t=2 s to compute the velocity at 4 seconds. You should get 62 m/s.
(e) Now use the cumtrapz command to numerically integrate the acceleration from 2 to 4 seconds cumulatively, to produce a change in velocity which is now a function of time. Add this to the initial velocity (180 km/h expressed in m/s) to get the velocity as a function of time.
(f) Plot velocity vs. time.
(g) Integrate the velocity vs. time using trapz to compute the total change in distance from 2 to 4 seconds. You should get 110.66… m.

Problem 5

Create a function that computes the velocity of a point on a rigid body if given the velocity of another point on the same rigid body, a rotation vector and the position vector relating the two points.



Recall that the formula to relate these two velocities is

$$\vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r}_{A/B}$$

Your function should have the form

va=rigid(vb,omega,rab)

You may find the matlab function *cross* helpful… it takes the cross product of two vectors.

Problem 6

An energy method for solving a dynamic system ended up generating the nonlinear set of algebraic equations:

$$\frac{1}{2}k\left(\frac{m_a g}{k}-x_a\right)^2+m_a g x_a-m_b g x_b+\frac{1}{2}m_a {v_a}^2+\frac{1}{2}m_b {v_b}^2=\frac{1}{2}k\left(\frac{m_a g}{k}\right)^2$$
$$\frac{1}{2}\left((3-x_a)^2+4\right)^{-1/2}\left(2(3-x_a)\right)v_a=v_b$$
$$\sqrt{13}=\sqrt{(3-x_a)^2+4}+x_b$$

Unknowns: x_a , v_a, v_b
Given: m_a=*90 kg*, m_b=*135 kg*, *k=300 N/m*,*g=9.8 m/s*^{2}, x_b=*1 m*

Where the first equation comes from conservation of energy, and the other two are geometric constraints. Set up a function which defines this system, and use the *fsolve* function to find a solution. Physically, it is clear that x_a, v_a and v_b should all be positive, so choose an initial guess that converges to an all positive solution.

Problem 7

The van Der Pol oscillator is a classic nonlinear ODE:

$$\ddot{x}-a\left(1-x^2\right)\dot{x}+\omega^2 x=0$$

This system has negative damping for small motions, and positive damping for large motions. So there is some limit cycle where the oscillator stabilizes. Set up the vanDerPol oscillator as a nonlinear ODE in Matlab, and solve using *ode45* for a=1, ω=10, with initial conditions *x(t=0)=1* and *dx/dt (t=0) =0.* Solve the equation over the range t=0 to 10.